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NEWER CONCEPTS OF THE PITCH, THE LOUDNESS AND THE TIMBRE OF MUSICAL TONES.*

BY

HARVEY FLETCHER, Ph.D.

When the subject "Newer Concepts of the Pitch, the Loudness and the Timbre of Musical Tones" was announced, I am sure most of you asked yourselves the question "What are the older concepts?" and thought of some of the simple statements given in most textbooks on physics. For example, here is a quotation from such a textbook which is widely used: "Pitch depends upon the length of the sound wave, loudness upon its amplitude, and quality or timbre upon the form of the complex wave."

Most of you no doubt would agree with this statement. According to the newer concepts, this statement is still true but its implications are wrong, for I could say with equal verity that pitch depends upon the amplitude of vibration, loudness upon the form of the wave, and timbre upon the length of the wave. Or I could say truly that the pitch depends upon the form of the wave, loudness upon the length of the wave, and timbre upon the amplitude of the wave. Any of these three statements would not be sufficiently inclusive. Let us then examine how the three psychological characteristics of a musical tone, namely, pitch, loudness and

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timbre, depend upon the physical properties of the sound wave producing them.

A sound wave capable of producing a simple musical tone can be described by giving the intensity, the frequency, and the phase of vibration of each component at the position where the observer is to listen. It has been found by experimental trial that the auditory sensation produced by such a tone is independent of the phase of vibration except for very intense sounds. Since changing the phase of the components produces large changes upon the wave form, it is evidently wrong to say that the timbre or tone color of musical sounds is completely determined by the wave form, or for that matter, that it changes as the wave form changes. It may or may not change depending upon how this wave form change is accomplished. Timbre is more nearly correlated to what I choose to call overtone structure. The overtone structure gives the relative intensities and frequencies of the components but not the phases. For these reasons, it is convenient to define the sound wave producing the simple musical tone by giving its total intensity, its fundamental frequency, and its overtone structure, near the ear of the listener.

The intensity can be determined by measuring the r.m.s. value of the pressure variation using a sound meter. The intensity then is proportional to the square of the r.m.s. value of the pressure thus determined. The frequency may be determined by any of the familiar methods. The overtone structure can be determined by using a suitable harmonic analyzer. The following example will illustrate the results obtained by such measurements.

A baritone sings the vowel "Ah" at a pitch A below middle C with a moderately loud voice. At all listener's positions in a medium-sized concert hall the frequency remains approximately constant at 147 c.p.s. The r.m.s. pressure which is produced will be between 1/10 and 1 bar (dynes per square centimeter), depending upon the position. The overtone structure will also vary somewhat from one position to another but the following one will be typical.

$$\frac{2}{0.38}, \frac{3}{0.06}, \frac{4}{0.01}, \frac{5}{0.12}, \frac{6}{0.12}, \frac{7}{0.13}, \frac{8}{0.04}, \frac{9}{0.01}, \frac{10}{0.01}, \frac{11}{0.002}. \quad (1)$$

The number above the bar is the number of the harmonic and the one below the relative intensity of this harmonic expressed as a fraction of the intensity of the fundamental. For example, for this tone the second harmonic had an intensity of 0.38 that of the fundamental. The third and fourth harmonics were very weak while the fifth, sixth and seventh were stronger. Beyond the seventh, the harmonics become continuously weaker as they become higher.

Having these physical concepts in mind, let us consider the nature of the auditory sensations called, respectively, loudness, pitch and timbre. Loudness is the magnitude of the sensation. However, its quantitative use depends upon finding a method of measuring it and of describing a scale for representing it. Since we cannot put a meter into the brain [1] and actually measure the magnitude of the nerve impulses reaching the auditory center, we must rely upon the judgment of individuals regarding the sensations which they receive. For this reason a reference tone has been adopted for making loudness comparisons. This tone has a vibration frequency of 1000 c.p.s., and is free from overtones. For experimental purposes it is arranged so that any intensity throughout the audible range can be produced. This intensity can be determined by measuring the r.m.s. pressure at the position where the observer is to listen. Apparatus for producing this reference tone is usually calibrated in decibels, a unit which is related to pressure by the equation

$$\text{db} = 20 \log \frac{(\text{Pressure in bars})}{0.0002},$$

the reference pressure 0.0002 bars being near the threshold of hearing. Since most of you are more familiar with the pressure units, I will express the intensities as the r.m.s. pressure variation in bars.

A true loudness scale must be such that when the number of units on this scale is doubled, the magnitude of the sensation as experienced by typical observers will be doubled, or when trebled the loudness sensation is trebled, etc. The existence of such a scale is dependent upon the supposition that con-

[1] Such a procedure is possible when experimenting with animals.

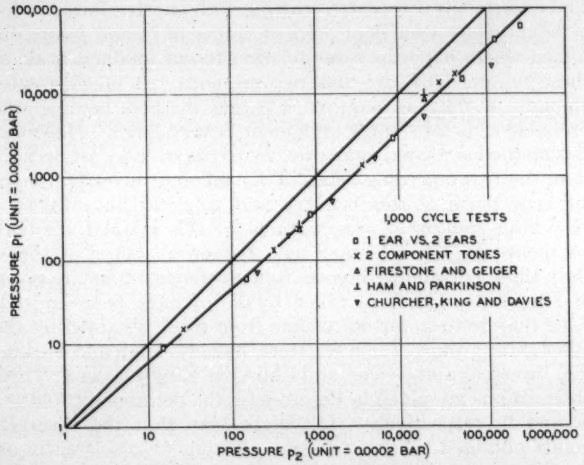
sistent judgment tests of loudness can be obtained. Experimental tests indicate that such judgments are sufficiently consistent to warrant choosing such a scale. To find the relation between the intensity of the reference tone and its loudness on the proposed new scale, various types of experiments were performed by investigators in our laboratory and elsewhere.

The first type of experiment is concerned with the comparison of the loudness experienced by a typical observer when listening first with both ears and then with only one ear. How much is the loudness decreased by such a procedure? If loudness is proportional to the total nerve energy sent to the brain, and the two ears have the same sensitivity, then it follows that the loudness is decreased to one-half. Let us call p_1 the r.m.s. pressure at the listener's position while listening with both ears and let N_1 be the corresponding number giving the loudness on our proposed scale. If one ear is closed the r.m.s. sound pressure must be increased to some value p_2 before the loudness appears to be the same as that produced when listening with both ears to an r.m.s. pressure p_1. Now we will let our typical observer listen with both ears to the pressure p_2. The loudness experienced must be $2N_1$. If we now repeat the experiment listening with both ears first to the pressure p_2 and then finding the corresponding pressure p_4 which gives the same loudness listening with one ear, p_4 will then correspond to $4N_1$. And so we might continue throughout the entire intensity range. If now we choose N_1 arbitrarily equal to some convenient number, we will have a set of corresponding values of pressure and loudness from which we can draw a curve giving the relation which we were seeking. The experiments as outlined could not be carried out because our typical observer was hypothetical. To obtain results corresponding to that which would be obtained by such a typical observer, one must deal with a large group of observers taking the average value as that corresponding to such an observer. For this reason, pairs of pressures p_1 and p_2 were obtained starting with p_1 at various pressures which were unrelated but at different intensity regions throughout the audible range. The points indicated by □ in Figure 1 are such observed points. The other points in this figure will be explained later.

A solid curve is drawn to give the best fit of the observed points. From this curve the proper values of p_1, p_2, p_4, p_8, etc., can be obtained to build the scale in the manner described above.

In the second type of experiment the following procedure is followed. A 1000-cycle reference tone is balanced against a pure tone having a frequency far removed from 1000 until they are equally loud. Then the two tones are combined to form a complex tone having two components. Let p_1 be the

FIG. 1.



pressure due to the 1000-cycle component in this combinations The reference tone is adjusted to pressure p_2 until it sound-equally loud to the two-component tone. Since the com. ponents were adjusted to produce equal loudness, each is producing the same nerve stimulation and hence when they act together they will produce twice the loudness. This is true provided in the mechanism of hearing the two sets of nerves do not interfere with each other. It can be shown by experiments on the masking of one tone by another that such

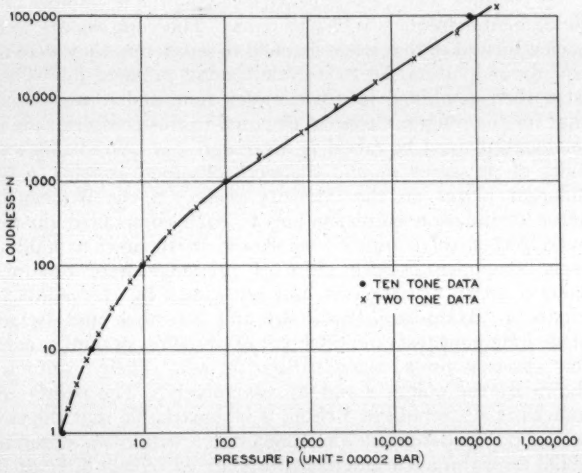
interference is negligibly small when their frequencies are widely separated. Consequently, balancing such a two-component tone against the reference tone should give pairs of values of p_1 and p_2 which can be used in the same way as indicated by the one vs. two-ear method described above. Such observed points are shown by the points x in Figure 1.

In the third type of experiment a typical observer is asked to listen to the reference tone, first at an intensity corresponding to a given pressure p_1 and then to the same tone when its intensity is raised to that corresponding to a pressure p_2 such that he indicates that the loudness has been doubled. The experiment is repeated starting with another initial value of p_1 and thus a series of pairs of values of p_1 and p_2 are obtained which have the same relation to our loudness scale as those obtained by the first two methods. A much greater amount of data is required by this method because the probable error of a single judgment is very large. However, this method is related more directly to the scale we are seeking than the two preceding ones. Four different investigators in different parts of this country and England have taken a very large number of observations by this method not only on the 1000-cycle reference tone but on all types of tones. Only those pertaining directly to the reference tone are given in Figure 1. The data taken by Laird have been omitted since they were so far out of line from those obtained by the other three groups of investigators, namely, Ham and Parkinson, Firestone and Geiger, and Churcher-King-Davies. Their observations are given in Figure 1 by the points indicated $\perp$, Δ, and ∇, respectively. It will be seen that the observed points obtained by these diverse methods fit consistently on one curve, particularly well in the range of loudness usually experienced in everyday life. It was sufficiently consistent to justify choosing the resulting loudness scale as a true loudness scale.

Using this curve in Figure 1 pairs of values of p_1 and p_2 were taken to construct the curve of Figure 2 in the same manner as described above for the first two types of experiment. The crosses show the values used. A value of $N = 1$ was taken arbitrarily for a pressure 1 that is at the threshold of hearing. It will be seen that these crosses are

at positions 1, 2, 4, 8, 16, 32, 64, 128, etc., on the loudness axis. It was found that for such a choice a pressure ratio of 100 corresponded very closely to a loudness N of 1000. For this reason the pair of values $p = 100$ and $N = 1000$ was taken as the starting point. The points below this were obtained by halving and those above by doubling the loudness values.

FIG. 2.



Loudness scale.

DEMONSTRATION I.

To illustrate the first method of obtaining pairs of values of p_1 and p_2 a 1000-cycle tone was produced at a loud level. Those in the audience were asked to listen first with both ears and then with one ear. This was accomplished by closing one ear with the finger. This was repeated two or three times using first one ear then the other until the increase in loudness was noted. The experiment was repeated except this time the pressure for the tone listened to by one ear was increased three-fold. A vote was taken by show of hands on which condition was louder. The audience was about equally divided showing this value to be approximately the average.

This same scale can be developed from data obtained by balancing a three-component tone against the reference tone or by judging when a three-fold increase in loudness has been produced. Or we could use any other number besides 2 or 3 and should get the same scale if it is a real loudness scale. An extreme case would be to use a ten-component tone and use judgment tests for 1/10 and ten times as loud. Tests of this kind have been made which I will now describe.

A complex tone [2] was generated having ten harmonics of a fundamental frequency of 530 c.p.s. The components were each adjusted so that when listened to separately they were all equally as loud as the 1000-cycle tone of pressure p_1. They were then combined into a complex tone and it was found that for the reference tone to be equal to this complex tone in loudness, it must be raised to a pressure p_{10}. So in this way pairs of pressures p_1 and p_{10} were obtained starting p_1 at different places on the intensity scale. If the patches of nerve stimulation corresponding to each component do not overlap then the complex tone should be 10 times as loud as each component. Such pairs of pressures were obtained using a group of observers and are shown by the points in Figure 3. Ham and Parkinson and Firestone and Geiger made judgment tests to find a pair of pressures p_{10} and p_1 such that the tone for p_1 was 1/10 that for p_{10}. Their results are shown by the points x and Δ, respectively. The points are somewhat scattered but I think it is remarkable that the two sets of points determine the single curve within the observational error, for such judgment tests are extremely difficult to make. It is surprising that the agreement is as good as indicated. The judgment tests and the ten tone balance tests were all made in different laboratories and before any loudness scale of the type discussed here was constructed.

Now the curve of Figure 3 can be used to construct a loudness scale. Starting with $p = 100$ and $N = 1000$ as before, we find $p = 3500$ corresponds to $N = 10{,}000$; $p = 80{,}000$ to $N = 100{,}000$; $p = 10$ corresponds to $N = 100$; $p = 2.5$ corresponds to $N = 10$; and $p = 1.2$ corresponds to $N = 1$. These are the circles on the curve of Figure 2. It is seen

[2] The description of this generator is given in *Jr. Acoustical Soc.*, Oct. 1933, Vol. V, No. 2, pp. 82–108.

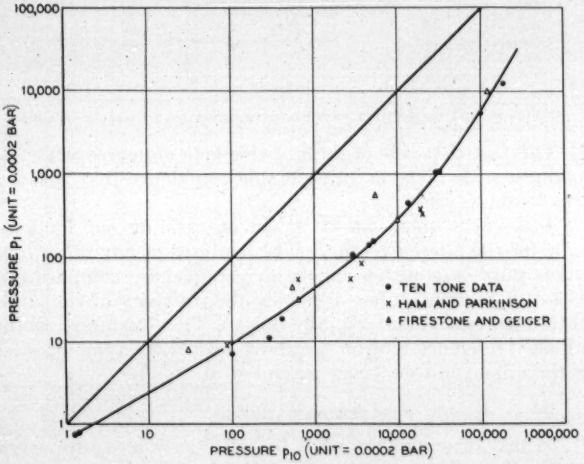
that they do indeed determine the same curve. That portion of the loudness curve of Figure 2 above $p = 100$ is drawn straight and gives a good fit of the observed points. Since this is a logarithmic plot this means that in this range

$$\log N = A + B \log p.$$

Determining the constants A and B from the curve it is found that above $p = 100$ the simple relation exists

$$3 \log N = 2 \log p + 5$$

FIG. 3.



or

$$N = 46p^{2/3} = 46\sqrt[3]{I},$$

where the pressure p and the intensity I are measured in terms of threshold values. This relation will then enable one to calculate the pressure ratios of any fractional reduction in loudness. Let p_0 be the initial pressure and p the pressure to reduce the loudness to the fraction F. Then

$$\frac{N}{N_0} = F = \sqrt[3]{\left(\frac{p}{p_0}\right)^2} \quad \text{or} \quad \log F = \frac{(S - S_0)}{30},$$

where S and S_0 are decibels above threshold corresponding to p and p_0 which values are the ones recorded by most investigators. For example, here is a set by Ham and Parkinson.

TABLE I.

$p_0 = 20{,}000$.			$p_0 = 630$.		
p.	Obs. F.	$\sqrt[3]{\left(\frac{p}{p_0}\right)^3}$.	p.	Obs. F.	$\sqrt[3]{\left(\frac{p}{p_0}\right)^2}$.
20,000	1.00	1.00	630	1.00	1.00
13,200	.68	.75	500	.93	.85
10,000	.53	.59	380	.75	.71
6,300	.41	.45	280	.55	.58
3,600	.26	.32	200	.41	.47
2,000	.20	.21	112	.25	.32
630	.13	.10	63*	.11	.18
200	.08	.05			

* This was calculated from loudness curve rather than from the formula.

The fact that sets of data of this type do agree with this loudness scale gives us considerable confidence that this is a true loudness scale.

This curve then can be taken as defining our loudness scale for the reference tone. The loudness of any other tone either pure or complex can be determined by comparing it with this reference tone, adjusting the intensity of the latter until the two sound equally loud. The loudness of the adjusted reference tone on the above scale is then the loudness of the unknown tone being measured.

DEMONSTRATION II.

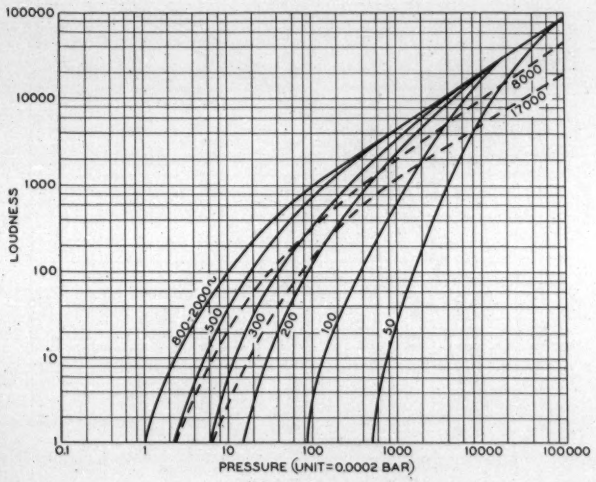
So that these numbers could be better associated with actual sounds the reference tone was produced at loudness values of approximately 100, 1000, 10,000, and 100,000, and was sounded two or three times in succession.

In this way a large number of tones having various fundamental frequencies and overtone structures have been measured to determine their loudness. In the next figure the results of such measurements for pure tones are given. The abscissæ give the r.m.s. pressures near the ear of the listener and the ordinates give the loudness of the tone as judged by a number of observers. It will be realized, of course, that

these judgments varied over a considerable range and these curves give only the average result. From the mean deviations the probable error of this average value of the loudness number was calculated to be from 10 to 20 per cent.

It will be seen that tones having frequencies in the range from 800 to 2000 c.p.s. have approximately the same loudness when the r.m.s. pressures are the same. There is only a small difference even for tones having frequencies as high as

FIG. 4.



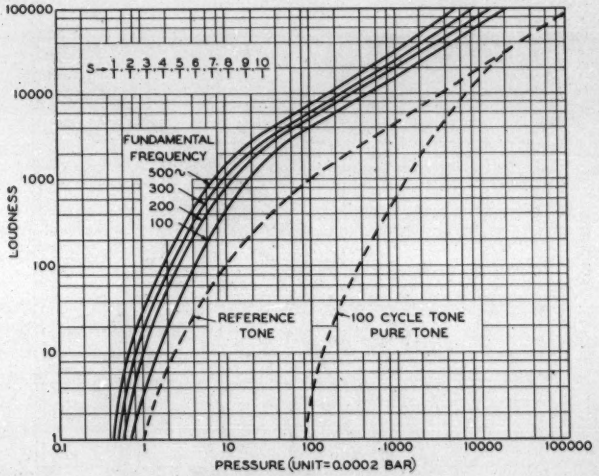
Pure tones.

8000 c.p.s. For higher frequencies than this the loudness decreases rapidly as the frequency increases. It will be seen that for equal r.m.s. pressures the low frequencies are considerably less loud. For example, compare a pure tone of 100 c.p.s. with one having 1000 c.p.s. When they both have a r.m.s. pressure of 0.02 bar, the former has a loudness of 3 and the latter a loudness of 1000; however, for a common pressure of 2 bars they have approximately the same loudness of 17,000. This effect was demonstrated.

DEMONSTRATION III.

A 100-cycle tone and a 1000-cycle tone were adjusted to sound equally loud when both were at a high intensity level. The attenuation before the loud speaker was then gradually increased so that both tones were decreased 3000-fold. Before this point was reached the 100-cycle tone was inaudible whereas the 1000-cycle tone was still reasonably loud at the end.

FIG. 5.



Tones having ten components equally intense.

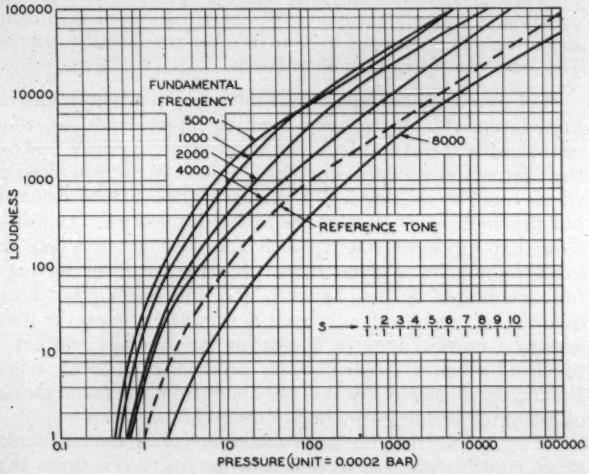
It is this property of the low pitched tones that makes the bass in a musical selection much more prominent when it is heard at close range or reproduced very loudly than when heard at a distance or reproduced very softly.

Now let us see what effect adding an overtone structure to this 100-cycle tone will have upon the loudness. Let us consider the overtone structure.

$$S \rightarrow \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ = & = & = & = & = & = & = & = & = & = \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}.$$

This represents a tone with *ten harmonics* all equally intense including the fundamental. The set of curves in Figures 5 and 6 represents the relation between loudness and intensity for tones having such an overtone structure. The numbers attached to each curve give the fundamental frequency of vibration. In Figure 5 the 1000-cycle pure tone and the 100-cycle pure tone are included for comparison purposes. The curve for $f_1 = 500$ c.p.s. is repeated in Figure 6 so that the

FIG. 6.



Tones having ten components equally intense.

other curves may be compared to it. It is seen that changing the overtone structure from no overtones to 9 equally intense ones has increased the loudness from 20 to 6000 for this particular tone having the fundamental frequency of 100 c.p.s. and a r.m.s. pressure of 0.04 bar. Another experiment illustrated this. First the 100-cycle pure tone was adjusted to produce approximately 1/10 bar pressure variation near the ears. Next a second 100-cycle tone was sounded to

which the first 9 harmonics had been added and also the intensity of the fundamental and all the harmonics had been adjusted to be only 1/10 of that corresponding to the first tone. In other words, it was arranged so that the intensity of the complex tone thus formed and also its fundamental frequency were the same as those of the pure 100-cycle tone just heard. The audience noticed the very large increase in loudness. There was also a marked quality difference.

DEMONSTRATION IV.

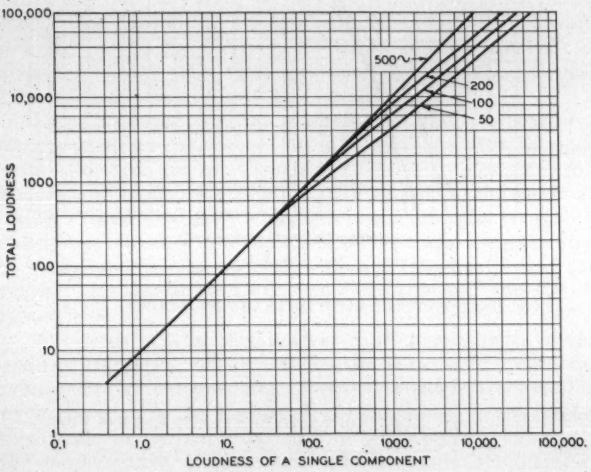
First the 100-cycle tone was sounded so as to give a pressure variation of about 1/10 dyne. The harmonics were then added and the amplifier changed so as to decrease the intensity ten-fold. The tones were listened to alternately.

Let me emphasize again that these two tones both had the same intensity and frequency and, therefore, could be transmitted with the same electrical or mechanical power and yet the loudness was vastly different. Increases are produced on all the tones by making such a change in the overtone structure but this increase is least at the higher intensities. It will be seen that for tones having such an overtone structure the loudest tones for a given r.m.s. pressure are those having a fundamental frequency between 400 and 800 c.p.s. These quantitative results show why it is easy to increase the loudness of a musical tone by increasing its overtone content, a practice which is common in producing musical tones. Practically all the loudness of the tones from the piano strings of low pitch is due to the higher overtones.

It is obvious that the 500-cycle tone having ten harmonics all equally intense will have a total intensity ten times that of each component. Also, since for this tone the components are sufficiently separated in frequency so that each one stimulates a separate patch of nerves on the end organ of hearing, and for these frequencies equally intense components mean approximately equally loud components, its loudness will be ten times that due to each component when listened to separately. However, as the frequency of separation becomes smaller these stimulated patches overlap and the total loudness from such a ten-component tone will always be less than ten times that produced by a single component acting alone.

This is illustrated by the curves in Figure 7. The results represented by these curves are for ten component tones, the intensity of them being adjusted such that when the components are listened to separately they sound equally loud. The numbers on the curves give the fundamental frequency in c.p.s. For loudness values below about 600, the combined loudness of the ten components is ten times that of each component. This is also true for higher loudness values if the

FIG. 7.

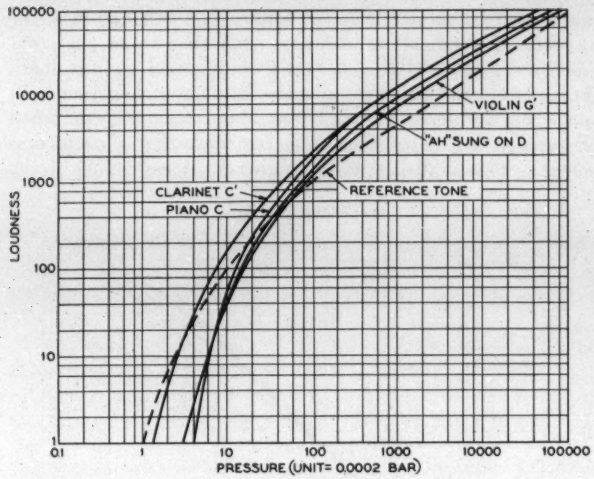


Tones having ten equally loud components.

frequency separation between the components is 500 cycles or greater. For a frequency separation greater than 1000, it is difficult to make the higher components equally loud as the ear becomes insensitive for these very high frequencies.

In Figure 8 there is a third set of curves which shows the relation between loudness and r.m.s. pressure for four tones having different overtone structures corresponding to the musical instruments noted on the curves.

FIG. 8.



Musical tones.

These four overtone structures are as follows:

TABLE II.

Piano.....	$\frac{1}{1}$	$\frac{2}{0.45}$	$\frac{3}{0.1}$	$\frac{4}{0.12}$	$\frac{5}{0.02}$	$\frac{6}{0.3}$	$\frac{7}{0.02}$	$\frac{8}{0.09}$	$\frac{9}{0.04}$	$\frac{10}{0.04}$	$\frac{11}{0.03}$	—
Clarinet.....	$\frac{1}{1}$	0	$\frac{3}{0.34}$	0	$\frac{5}{0.01}$	$\frac{6}{0.003}$	$\frac{7}{0.01}$	$\frac{8}{0.12}$	$\frac{9}{0.17}$	$\frac{10}{0.24}$	$\frac{11}{0.02}$	$\frac{12}{0.01}$
Violin.....	$\frac{1}{1}$	$\frac{2}{0.18}$	$\frac{3}{0.02}$	$\frac{4}{0.003}$	$\frac{5}{0.01}$	$\frac{6}{0.002}$						
Voice ah.....	$\frac{1}{1}$	$\frac{2}{0.38}$	$\frac{3}{0.06}$	$\frac{4}{0.01}$	$\frac{5}{0.12}$	$\frac{6}{0.12}$	$\frac{7}{0.14}$	$\frac{8}{0.04}$	$\frac{9}{0.01}$	$\frac{10}{0.01}$	$\frac{11}{0.002}$	

It is seen that the variation of loudness with r.m.s. pressure is approximately the same for these tones as for the reference 1000-cycle tone. However, at the higher intensities the loudness for these musical tones is from $1\frac{1}{2}$ to 3 times greater than that corresponding to the reference tone. It was found that the experimental results could be represented by the simple formula

$$N = \sum_{k=1}^{k=m} b_k N_k,$$

where N is the number representing the loudness of the complex tone, N_k the number representing the loudness of each component tone listened to separately and b_k is a number between 0 and 1 depending upon the intensities and frequencies of the two neighboring components to the k^{th}, namely, the $(k-1)^{th}$ and the $(k+1)^{th}$. The relationship giving b is somewhat complicated and is fully treated in the paper mentioned below. In many cases b is unity so that if the loudness numbers corresponding to each component are added together they will give the total loudness of the complex tone. One knows that the loudness thus obtained is always too high while the loudness of the loudest component is always too low. This then will serve to locate the loudness sufficiently accurately for a great many purposes.

Enough has been given to show that the loudness of a musical tone as experienced by a typical observer depends upon the three physical quantities, namely, intensity, frequency and overtone structure. Nevertheless, the loudness changes more rapidly with changes in the intensity than with changes in the other two variables. Those who are interested in studying this relationship in more detail should read the paper[3] "Loudness, Its Definition, Measurement and Calculation."

The pitch of a musical tone is that characteristic of the auditory sensation which enables the listener to locate the position of the tone on a musical scale. It is probably related to the position of the stimulated nerve endings on the basilar membrane in the ear. Because of its subjective character the measurement of pitch can only be made by judgment tests. Since individuals vary in such judgment tests, it is necessary to use several listeners and average the results in order to get something that may be called characteristic of a "typical" listener.

As was the case for loudness a greater accuracy can be obtained for pitch determinations by choosing a reference standard. The reasons why the need for such a reference tone has not been apparent is due to the general acceptance of the false idea that the pitch is definitely determined by the frequency of vibration and that consequently a measurement of

[3] Fletcher & Munson, *Journal of Acous. Soc.*, October, 1933.

the latter determines the former. As we shall see, experiments indicate that this one-to-one relationship does not hold and indeed for some cases of complex tones the frequency of vibration is indeterminate since such a tone is composed of a number of non-harmonic components. For most musical tones, however, the components have frequencies which are multiples of a fundamental, and for such tones this fundamental frequency may be considered as the frequency corresponding to the tone. For these reasons, it is necessary to define a reference tone and a scale for determining the pitch of a musical tone.

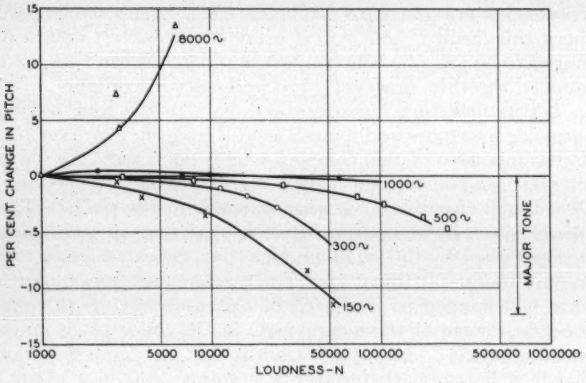
For this purpose a tone is chosen having a constant loudness of 1000 and capable of being varied in frequency throughout the audible range. We will arbitrarily define the pitch of our reference tone by its frequency of vibration. The pitch of any other tone pure or complex can be determined by comparing it with this chosen reference tone by adjusting the latter until the two are judged to have the same pitch by a group of observers. For example, such a comparison shows that a tone having a vibration frequency of 200 c.p.s. and a loudness of 100,000 has a pitch of 180 c.p.s.; that is, its pitch lowers 20 c.p.s. as its loudness is increased from 1000 to 100,000.

This phenomenon of pitch change with intensity change has been known for some time but no quantitative results were available until recently. The results are not yet sufficient to cover the field. Typical results obtained with pure tones are given in Figure 9. These curves were taken from data recently published by Stevens.[4] The ordinates give the per cent. change in pitch when the loudness is changed from 1000 to the value shown by the abscissa. The double arrows show the magnitude of a major tone on this scale. It will be seen that for large changes in loudness the pitch may change as much as a full musical tone without any change in frequency.

Although the data for tones having frequencies below 100 cycles are not very consistent, they indicate that the pitch change becomes smaller as the frequency becomes lower. The pitch changes for tones having frequencies above 1000

[4] *Journal Acoust. Soc.*, Jan., 1935.

FIG. 9.



Pitch change as a function of frequency and loudness.

cycles are small and for very high frequencies the pitch increases with increases in the intensity.

DEMONSTRATION V.

To illustrate this change in pitch with increase in intensity, a demonstration was tried which was somewhat difficult because of the standing waves produced in the room making the intensity at some positions very much greater than at others. When the tone was sounded the audience was asked to first listen with both ears; then close both ear canals as tightly as possible either by means of the fingers or by clasping the hands tightly over the ears. This was repeated two or three times and the change in pitch noted. The experiment was difficult due to the standing waves but almost everybody reported that they noticed the change.

The following comparison is interesting. Let *A* be a tone defined by frequency = 150 c.p.s. and loudness = 1000; *B*, a tone defined by frequency = 300 c.p.s. and loudness = 1000; *C*, a tone defined by frequency = 168 c.p.s. and loudness = 50,000; and *D* a tone defined by frequency = 318 c.p.s. and loudness = 50,000. Then it will be seen from Figure 9 that the tone *C* has the same pitch as *A* and the tone *D* the same as *B*. Since *A* and *B* are just one octave apart in pitch, then

C and *D* must be one octave apart in pitch although their frequencies are 168 c.p.s. and 312 c.p.s., respectively. To check this result *C* and *D* were sounded alternately and were judged to be just one octave apart in pitch. When they were sounded together, however, they were very discordant.

Let us now turn our attention again to the tone having harmonic overtones and a fundamental frequency of 100 c.p.s. As the intensity of this complex tone is decreased, the lower pitched components go below the threshold of hearing. Under such circumstances what happens to the pitch? The answer given by comparison tests is that the pitch does not change except by the small amount to be expected from tests on pure tones. If this is true, one then immediately inquires what will happen to the pitch of this tone having 10 components if some of the components in the lower pitch range are eliminated. The answer again is that the pitch does not change. This remarkable fact was pointed out in a paper [5] about 10 years ago. The experiments at that time were made with tones produced by a telephone receiver held to the ear. Recently, these experiments have been repeated and extended by means of the loud speakers used in the Auditory Perspective Demonstration.

The 10-tone generator employed in the demonstrations was used for generating tones having frequencies of 100, 200, 300, 400, 500, 600, 700, 800, 900 and 1000 c.p.s. Results were found to be similar to those found earlier when using head receivers. If any four consecutive tones were sounded the pitch obtained by judgment tests using the reference tone for pitch was found to be 100 c.p.s. If the tones 400, 600, 800, 1000 were sounded simultaneously, the pitch would be 200 c.p.s. To further emphasize these pitch changes some experiments were performed using various combinations of the harmonics of 100 cycles.

DEMONSTRATION VI.

A. Components having 700, 800, 900, 1000 cycles were sounded together. The pitch corresponded to 100 cycles as judged by a prominent musician who came from the audience and picked out the pitch on a piano which was available.

[5] *Physical Rev.*, Vol. 23, No. 3.

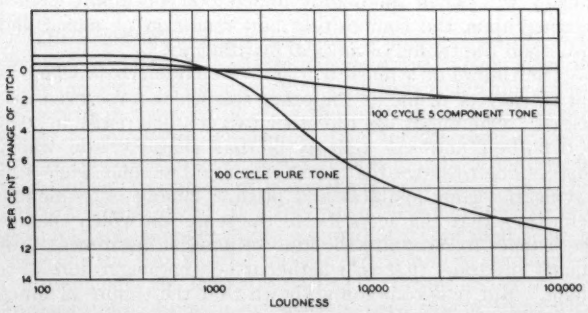
B. The components 400, 500, 600, 700, 800, 900, 1000 were sounded and the odd harmonics were eliminated. The pitch then was found to be 200, that is, it jumped up an octave.

C. The 400 and 500, the 600 and 700, the 800 and 900 tones were sounded in pairs. In each case the audience reported that they could hear the 100-cycle subjective tone.

A particularly interesting combination was tried. When the tones corresponding to frequencies 400, 600, 800, and 1000 were sounded together, the pitch was found to be 200 cycles per second. To this combination, the frequencies of 500, 700 and 900 c.p.s. were added; the pitch of the combination then was found to drop to 100 cycles per second. That illustrated this peculiar phenomenon, that the addition to a certain complex tone of three components all of which have frequencies above 500 c.p.s., produced a lowering in pitch in the complex tone from 200 to 100 c.p.s.

A complex tone whose components have a harmonic series of frequencies has a very definite pitch. This pitch corresponds approximately but not exactly to that of the fundamental of the series. For example, as shown in Figure 10, a

FIG. 10.



tone having five such components with a fundamental frequency of 200 c.p.s. changed downward in pitch only 2 per cent. as the loudness was raised from 1000 to 88,000. This corresponds approximately to the change in pitch of any one of the upper three components if it were sounding alone. On the other hand, it was seen that a 200-cycle pure tone

changed 10 per cent. or 5 times as much as the complex tone for this same loudness change. Therefore, the pitch of the complex tone at the higher loudness is 8 per cent. higher than that of the pure 200-cycle tone. When these tones were sounded alternately on this high loudness level, it was found that the pure tone was actually lower in pitch by approximately the amount indicated by the above figures. However, if the two are sounded together they are not discordant for under such circumstances a new tone different from either is formed. In this new tone the fundamental is strengthened and the pitch of the resulting combination is slightly lowered so that its pitch is about 3 per cent. below a pitch of 200 c.p.s. So one cannot conclude since two tones sounding separately are slightly different in pitch, that when they are sounding together they will be discordant. They may or may not be so. Conversely two tones sounded alternately may have the same pitch but may be discordant when sounded together. A combination will be harmonious or not, depending upon the frequencies rather than on the pitch of the components. These facts concerning the relation between pitch and frequency will, when thoroughly understood, no doubt have a bearing upon the composition and rendition of music and also upon the tuning of musical instruments.

The timbre of a musical tone is a characteristic which is not so easy to define as the other two which have been discussed. One might use the other two characteristics in such a definition and say that it is that characteristic which enables one to judge that two tones are dissimilar while still having the same loudness and pitch. Timbre is frequently defined as that characteristic of the sensation which enables the listener to recognize the kind of musical instrument producing the tone, that is, whether it is a cornet, a flute or a violin. But it is commonly known that the timbre of tones coming from two different violins may be greatly different and we have no adequate language to express this difference. Such a language is greatly needed. Very little experimental work of scientific value has been done on timbre. It is true that considerable work has been done in attempts to find the over-tone structure of musical tones, but such work is very different from obtaining judgment tests under scientific

control of various types of musical tones. By analogy with what has been said on pitch and loudness it seems advisable to choose scales and a reference tone for making comparisons of timbre. It is well known that changes in overtone structure produce corresponding changes in the timbre but is not so well known that changes in loudness or pitch, without in any way changing the overtone structures, will also produce changes in timbre. For example, when a violin sound is reproduced over a high quality reproducing system in such a way that its overtone structure is unchanged but its intensity amplified so as to have a loudness level 10 or 20 db higher than the sound coming directly from the violin, then it no longer sounds like a violin. Certainly the timbre has been greatly changed. Consequently, in selecting a reference tone for timbre comparison, we must select apparatus so that the intensity, the frequency, and the overtone structure of the reference tone can be varied throughout the audible range. Such a tone generator would be very difficult to build. Some attempts have been made to construct such a generator but no one has succeeded in devising as versatile a one as is required.

The problem becomes much more simple if we limit the overtone structure to a simple harmonic one, which, after all, is the important type of overtone structure with which we are concerned in most music. At the present time such a generator for producing a reference tone for the comparison of timbre is being developed in our laboratory, and I understand that similar attempts are being made in some of the other research laboratories. It is only when these machines become available and judgment tests are made that we shall begin to trace out the complicated ways in which timbre is related to the frequency, the intensity and the overtone structure of a musical tone. Enough has already been said to show that timbre depends upon all three of these physical characteristics.

Thus we see that of the three characteristics of the sensation experienced by one listening to a musical tone, the loudness depends principally upon the intensity, but changes in the frequency or the overtone structure may sometimes produce large changes in loudness. Also, the pitch depends principally upon the frequency but changes in the intensity

produce small changes in the pitch and certain types of changes in the overtone structure may produce large changes in pitch. And finally, the timbre depends principally upon the overtone structure, but large changes in the intensity and the frequency also produce changes in the timbre.

These results can be at least partially explained by the mechanism of hearing. Although it is not my purpose to discuss this phase of the subject in any detail here, it may be of interest to mention the following: The peculiar phenomena of loudness are no doubt principally due to the ear acting as an analyzer separating the components having different frequencies in such a way that they produce stimulations on different sets of nerves, and also due to the variation with frequency of the vibrating parts of the hearing mechanism. It is the summation of these different stimuli that gives us the sensation of loudness. Except for very low or very high frequencies, the pitch is probably determined by the position of the nerves of maximum stimulation. The fact that the pitch changes with intensity changes is probably due to the following dynamical facts.

When a resonator is forced to vibrate at very high amplitudes, its resonant frequency is sometimes increased and sometimes decreased depending on the kind of resonator. Pendulums and tuning forks decrease while stretched strings increase their resonant frequencies for high amplitudes. For both classes of resonators, the restoring force is proportional to the strain for small amplitudes. For large amplitudes for the first class the restoring force is less and for the second class it is greater than it would be if this proportionality extended to the high amplitudes. This change in the restoring force accounts for the difference. Apparently the resonators in the ear act like stretched strings for those frequencies below 2000 c.p.s. and like tuning forks above that frequency. This would cause the stimulated spot on the basilar membrane to be shifted for increased amplitudes by amounts sufficient to explain the changes in pitch observed. It is an interesting problem of research to determine why the restoring forces of the resonators of the ear act in this way.

Timbre is probably associated with the form of the contours of stimulation in the two auditory sensation areas of the

brain. With these pictures in mind it can be seen why the total nerve energy sent to the brain, the position of maximum stimulation, and the form of the stimulation pattern in the brain all depend upon the three physical characteristics, namely, intensity, frequency and overtone structure, and consequently why these three physical characteristics must influence each of the three psychological characteristics, namely, loudness, pitch and timbre.

Simplified Cross Section Slicer.—DR. J. I. HARDY, senior fiber technologist of the Bureau of Animal Industry, has invented a small mechanical device which makes possible the rapid cross sectioning and clear study of the structure of hair, wool, mohair, fur, silk, cotton and other fibers, both natural and artificial. The new discovery enables the preparation of very thin cross sections ready for examination within ten minutes, whereas in the past it required several hours to obtain them. Sections, one-ten thousandth of an inch, can be cut without injury to the delicate internal structure. Moreover, such thin cross sections, even of seemingly opaque fibers, are sufficiently transparent for microscopic study.

Dr. Hardy has applied for a public patent on his device, which is about 3 inches in length and consists of three metal parts. A thin piece of metal contains a slot 0.0085 of an inch wide. A second thin flat piece of metal slides parallel to the first and pushes a short metal guide down the slot, pressing the fibers tightly together in a vertical position. The third part contains a small metal plunger on the end of a supporting screw for pushing the fibers ever so slightly through the slot. After the fibers are inserted and firmly secured in the slot, they are cut off on both sides of the holder with a safety razor blade. The plunger is then used to push the fibers from one side to the other according to the thickness of the cross section desired. A drop of celluloid is put on the projecting ends and this dries quickly without soaking into the fibers. The celluloid and fiber ends are cut off by the blade and the thin celluloid slice carrying the tiny disks of fibers is ready to be mounted on the microscope slide.

Many practical applications are forecast for this device. Information on the types of hair and fibers in various furs, fabrics, and industrial products can be supplied quickly and accurately. The invention should be valuable in livestock breeding by aiding stockmen in the development of animals with hair, wool, mohair, etc., of a kind in greatest demand in industry. It is particularly believed the new instrument will have far-reaching possibilities in the detection of crime. Cross sections of fiber clues may be studied directly through a microscope or by making microphotographs which may be filed away for further reference or sent to any part of the world for comparative study.

C.

VISIBILITY: ITS MEASUREMENT AND SIGNIFICANCE IN SEEING.

BY

MATTHEW LUCKIESH and FRANK K. MOSS,

Lighting Research Laboratory, General Electric Company, Nela Park, Cleveland.

Civilization is largely a world of seeing. Throughout its activities seeing is an intimately important factor in the efficiency and welfare of human beings. But, notwithstanding its overwhelming importance, seeing conditions are generally accepted as found and at best are largely a matter of empirical practice in which guesswork is prominent. Seeing specialists in the full sense do not exist. Instruments for analyzing seeing conditions and requirements have not been available. Researches in seeing have been meager compared with those in vision and, in fact, the development of an adequate science of seeing has only recently begun.[1] For many decades a science of vision has been developing but seeing involves much more than the tool—vision. It involves other tools such as light and lighting. In addition to these necessary tools, seeing is an activity of the human being operating as a seeing-machine. Therefore, an adequate study of factors and conditions of seeing must include the effects upon the human being which is doing the seeing. These effects are physiological and psychological and may even be morphological in extreme cases.

The foregoing briefly outlines the evolution of our thought and research throughout a quarter century. For the first decade we studied vision but introduced light and lighting as controlled variables. Eventually it became obvious that studies of vision could only yield knowledge of the threshold conditions.[2] Although it is essential to know the limitations of the visual sense, such knowledge pertains only to *barely seeing*. It does not include detection and measurement of possible hidden penalties which may be exacted of the human seeing-machine when working under the conditions of barely seeing. Thus eventually such conclusions led us to develop

new concepts of seeing as contrasted with the concept of vision as a tool. The conception of seeing as a human activity assumes that the effects of doing such work can be detected and measured throughout the body and even in mental attitude. Many years of research along the new lines have proved that seeing involves work for the body and mind and that effects of seeing conditions can be measured in terms of muscular fatigue, waste of energy, nervous muscular tension, heart-rate, etc. Thus an adequate science of seeing began to develop. Past research which had been devoted to the realm of barely seeing was extended in viewpoint by the new concepts and the course of research was directed toward the objectives of easy and easiest seeing.

On the new course much new knowledge pertaining to seeing conditions and requirements has already been revealed. Certain relationships have been established between threshold conditions for barely seeing and conditions for easy seeing. In some phases of this vast complexity we already have a fairly good idea of the ideal conditions. Certain valuable laws have been established for such tools as light and lighting. It is now known that intensities of illumination (footcandles) must be doubled in order to produce an obvious and significant improvement in seeing. In other words, footcandles must be increased in geometric ratio in order to improve seeing in arithmetic ratio. Interesting fundamental relationships between light and vision have been thoroughly established. As aids to human seeing-machines these two tools are at once complementary and alike in some respects. They are supplementary to each other in definite ways. Eye-glasses are helpful to many persons, but light and lighting are helpful in similar and dissimilar ways to all human seeing-machines.

The results of the new researches have particularly emphasized the fact that human beings are poor seeing-meters. They reveal a vast array of guesswork, inconsistency and ambiguity in dealing with seeing conditions. And, above all, they indicate the tragic tolls in human resources arising from the misconception that vision is seeing. Under the actual conditions of seeing, human beings can seldom "do what their eyes can do." For example, seeing a pedestrian or traffic aid may be easily accomplished while doing nothing

else. But to do this while performing other tasks such as driving an automobile and while much of the "sense capacity" is otherwise engaged is a far different matter. In other words, the new concepts of seeing as a *human* activity take into account *human* vagaries, which are generally unaccounted for in studies of vision.

The science of vision evolved largely through laboratory researches. The science of seeing must also be founded in this manner. It has been impracticable to take laboratory technique in vision out into the world of seeing. It is even more difficult to take the more difficult research technique in seeing out into practice. Civilization has not only suffered from the misconception that vision is seeing but also from a lack of simple portable instruments which could be substituted for guesswork and which were founded upon the facts and laws established through complex researches under the controlled conditions of the laboratory.

Guesswork is applied throughout the many highways and byways of civilized activities. Everywhere the applications of light and lighting have been devoid of a scientific basis. Researches in seeing are belatedly evolving a crude empirical lighting art into a science.[3] This is particularly interesting in a civilization highly scientific in many ways and made possible by modern artificial light. Throughout printing, which produces for human beings the most universal and common tasks of reading, scientific measurements of seeing are absent. Designers of type, advertising, traffic aids as well as the printing art, have in some cases used empirical methods but at best they are unwieldy or based upon laboratory data which are not readily transferable to actual conditions. Schoolbooks are sometimes chosen to be "easy on the eyes," but the choice is largely a matter of guesswork. Eye-glasses are scientifically prescribed, but out in the world of seeing the different requirements for old, young, subnormal and average adult eyes are generally ignored. There are aids available which can be applied if seeing conditions and requirements could be readily determined. Various visual tasks and types of work differ enormously in difficulty and in the resulting strain upon the human seeing-machine. Eligibility of workers for certain types of work cannot be completely determined by

eye-examinations. Measurements of their ability to see or to perform the actual task provide a safer answer. Appraising the influence of size, form, brightness, contrast and color of objects in seeing will continue to be largely a matter of guesswork in the world of seeing unless a simple dependable instrument is available. In legal cases visibility of automobiles, pedestrians, obstructions, hazards, and traffic aids is often important as well as a clear distinction between vision and seeing. In most cases only a crude presentation of the facts is made and even in the best prepared cases academic or theoretical knowledge is used in the absence of actual pertinent measurements.

Everywhere in a world in which seeing is complexly entwined with life and work, measurements of seeing conditions and requirements are needed. Therefore, throughout the course of many years of research in light, lighting, vision and seeing, one of our objectives has been to establish facts and laws which could eventually form the basis of a practicable instrument having rational scales of visibility and footcandles, for these are the most generally important and controllable factors in seeing. Measurements with such an instrument are not merely empirical but are soundly established upon the ability of average normal vision and of human seeing-machines. It has long been obvious that a human being can only determine threshold conditions by simple measurement. Therefore, we were forced to choose one or more of the factors of the visual threshold—size, contrast, brightness and time. Then the measurements had to be interpreted by means of appropriate scales into suitable knowledge. This paper presents such an instrument, the scientific foundation, the calibrations, and the factors for transferring the scale values to other bases when or if desired. It also reveals some of the complexity of visibility and some of the extensive research which establishes its significance in seeing. This Visibility Meter possesses the precision necessary for many researches in lighting, vision and seeing. It has already been used in this manner and as a research device it eliminates unwieldy technique in many cases. Besides simplifying many researches it greatly increases the practicability and possibility of research. The instrument has countless practical appli-

cations in a world of seeing which needs not only an adequate science of seeing but also measurements of seeing conditions and requirements. Only by such means can civilization be emancipated from the penalties and tragedies of ignorance and guesswork and the undesirable conditions arising therefrom. Seeing and human welfare [4] are becoming more intimately associated with each passing decade.

FUNDAMENTAL CONSIDERATIONS.

Since the perception of an object is an introspective phenomenon, it follows that the measurement of visibility cannot be reduced to an absolute basis in any case excepting by definition. Furthermore, such a measurement involves a threshold visual stimulus and hence it is significant in the world of seeing only in a relative sense unless suitable correlations are established. Although a series of visual measurements made under identical physical conditions may vary considerably in magnitude, individual observations may represent accurate appraisals of visibility under the biologic conditions which prevail at the moment of measurement. Therefore, such measurements may be regarded as reasonably reliable appraisals of a variable sensory process. In view of the fact that little control may be exercised over some of the causes of such variations, the physical conditions for seeing should be adequate to meet the requirements indicated by the most unfavorable observations by persons having reasonably normal vision. How far to go in the direction of satisfying subnormal vision is another matter.

A quantitative comparison of the effects upon seeing which are functions of the physical variables of the stimulus and the inherent biologic variables of the individual leads to the conclusion that at present the former are generally more significant than the latter in the world of seeing. There are relatively few individuals whose visual ability is of a threshold order for ordinary tasks yet nearly everyone must perform some tasks in the daytime and often many at night under near-threshold conditions due to physical aspects of the task. Even in cases involving important ocular deficiencies, it is obvious that the significance of these handicaps depends to a large extent upon the physical characteristics of the task to be

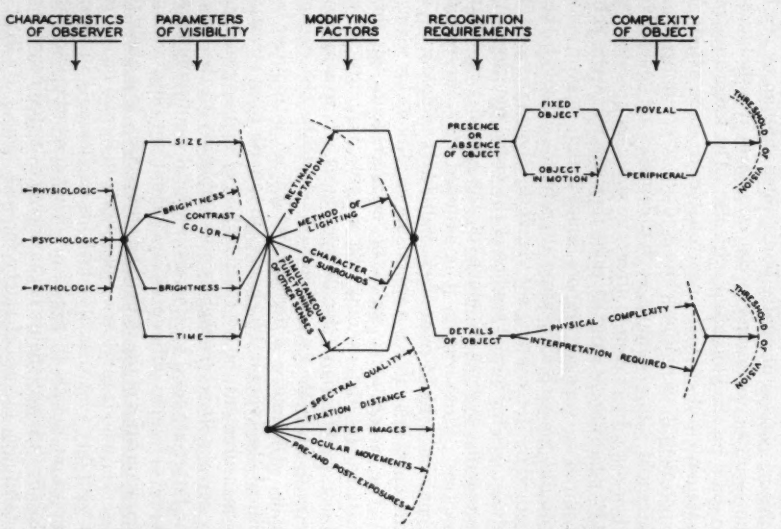
performed. Furthermore, it is the physical variables of the visual task which are universally controllable for the purpose of increasing the efficiency, comfort, and safety of human beings, and for conserving their resources and reducing penalties revealed by researches in seeing.

Some of the primarily important variables upon which the visibility of an object or the performance of a visual task depends are presented in Fig. 1. In fact, this diagram summarizes basic factors and assumptions involved in the actual measurement of relative visibility. Variables in this schema are indicated as radii of arcs; and the junction-points (solid circles) indicate the assumption of specific conditions for seeing in a particular case. Even with the comparatively few variables presented, it is obvious that the visual threshold is a complexity which is a function of almost innumerable combinations of these factors. Since certain arbitrary assumptions are required in measuring visibility in practice, the significance and character of these assumptions cannot be properly appraised without an adequate perspective of the entire complexity.

The appraisal of conditions for seeing, by the measurement of threshold visibility, obviously necessitates the use of socalled "normal" subjects in order to reduce the subject to an empirical basis for investigation. Very extensive statistical examinations of the data upon which the present discussion is based indicate that this fundamental requirement has been realized in fact. This phase of the complexity is represented by the first junction-point of Fig. 1. The selection of normal observers or subjects is rational for the purpose of determining the relationships between the physical variables of the stimulus and relative visibility. However, when such data are used as a basis for specifying the conditions for seeing upon which the efficiency, safety and welfare of human beings depend, the selective nature of the data should be recognized. This interpretive phase will involve the inclusion of "factors of safety" in order to provide adequately for anomalous cases and unusual conditions for seeing. The requirements for abnormal cases and conditions, as well as normal ones, may be appraised upon a quantitative basis by means of the new instrument described later.

FIG. 1.

VARIABLES IN THE MEASUREMENT OF VISIBILITY



Some of the more important variables upon which visibility depends. This diagram represents the chain of factors considered in developing rational scales for the Luckiesh-Moss Visibility Meter.

In addition to the characteristic accidental variations of visibility measurements, occasionally important systematic differences between individuals are obtained even though the subjects possess normal vision. Such differences are largely the result of variations in the criteria of threshold visibility adopted by the individual subjects. The magnitude of these systematic differences is controllable, to some extent, by the experimental technique employed in determining the visual threshold. For example, such differences are generally larger when the subject merely alters the stimulus until the "threshold" condition is reached than they are when the subject is required to correctly report some detail or position of the stimulus for each successive alteration. Although the latter method for determining the visual threshold possesses obvious advantages for formal studies of the visual functions, it is not necessarily typical of much of the visual performance in practice. Furthermore, it is assumed that an experimental technique which minimizes individual differences is not inherently desirable for appraising conditions for seeing.

The value of a series of measurements of threshold visibility for determining the relative visibility of various objects such as signs, signals, type-faces, advertising copy, etc. depends upon the technique with which the observations are made. Obviously, differences which are introduced by non-critical observation, by changes in criteria from measurement to measurement, or by the selection of inappropriate criteria by certain observers merely add ambiguity to the data when the latter are used to appraise physical conditions for seeing. However, such differences are significant in revealing the characteristics of the individual observers. It is likely that an observer who is careless or inaccurate in formal measurements of visibility can scarcely shed his carelessness in perceiving important details in practice. Therefore, the variability of visibility measurements is considered as a significant factor in determining factors of safety for such cases. At least something is learned of the vagaries, limitations and abilities of human seeing-machines.

Threshold size, contrast, brightness, and exposure-time are the fundamental parameters for expressing the relative visibility of different objects of regard.[5] For example, the

term "visual acuity" usually indicates that the size of the object of regard is the variable and its brightness, contrast, and exposure-time are constants in the determination of the visual threshold. The eyesight specialist usually appraises vision in terms of this variable. In everyday seeing contrast is far more important than is indicated by the general neglect of this factor. On streets and highways split-second seeing is often important and, therefore, the time required for seeing may seem to be a more appropriate criterion than does visual acuity. Since these are mutually related variables, it follows that the four variables are equally significant as fundamental criteria for the appraisal of relative threshold visibility. However, in the measurement of visibility in practice, each variable may possess unique advantages in certain specific cases. The selection of a particular variable for test-purposes is indicated by the second junction-point in Fig. 1.

In practice, it is usually neither convenient nor satisfactory to produce threshold conditions by reductions in either the size of the object of regard or in the time of exposure. Compared with these, brightness and brightness-contrast are fundamental factors which are readily alterable over wide ranges. For example, these two factors are varied simultaneously by the gradient filters of the visibility meter described in this paper. Since it would be inconvenient to express visibility in terms of two factors operating simultaneously as well as for other reasons, the instrument is calibrated in terms of threshold size.

Visibility is a function of innumerable combinations of many modifying factors, some of which are presented in Fig. 1. This complication usually prevents the determination of the relative visibility of different objects from the areal, photometric, and temporal characteristics of the objects and necessitates measurements under the specific conditions of each situation. For example, the relationships between the fundamental variables of the visual threshold, for a single object of simple geometric pattern, have been determined with precision for certain specific conditions of presenting the stimulus. However, it has also been shown that these relationships are greatly altered by other methods of presenting the same object.[6] Hence visibility data of a funda-

mental character, obtained under specific and usually arbitrary conditions, are applicable to other conditions only when the influence of modifying factors is known. It is seldom that this is completely fulfilled in practice. However, if actual visual situations are compared by means of a visibility meter possessing a rational foundation, these modifying factors are merely components of complex situations which are appraised in their entirety.

The variables included between the first and third major junction-points of Fig. 1 are chiefly those related to (1) the external physical world and (2) to the more or less direct psychophysiological effects produced by these physical conditions. In general, these represent the *non-volitional* elements of seeing. The phases of seeing which follow these are initiated by the individual himself. These volitional phases of seeing, in some cases, are modified by the physical characteristics of the objects of regard. *A fortiori*, this is the realm of seeing in which factors which contribute largely to the variability of visibility measurements are introduced. Uncertainty in seeing is a result of human vagaries. In this discussion of the characteristics of threshold visibility, vision is regarded as a tool and seeing is a result of a human being using this tool. Hence it may be assumed that the human seeing-machine does not invariably see as well as the eyes or the visual processes would theoretically permit. In other words, limitations of the human part of the human seeing-machine usually prevent the latter from "doing as well as his eyes can do." This fact is too generally ignored in everyday seeing. Physiologic fluctuations in the visual mechanism and psychologic fluctuations in attention lower the efficiency of the visual sense and introduce uncertainty as to the exact characteristics of the visual threshold. In specific cases, the degree of uncertainty may be estimated by biometric methods when extended series of measurements are available. For a specific human being under specific conditions the ability of the visual tool is comparatively constant but seeing is always elastic.

In some situations, the recognition of the presence or absence of an object is the primary visual requirement, but usually recognition of the object is necessary. In the latter case certain distinguishing details of the object must be

perceived. Obviously, the physical characteristics of the threshold stimulus are somewhat different in the two cases. These are related to the certainty with which the object of regard is seen; and the highest *measurable* degree of visibility is obtained when the object is seen correctly each time it is presented. This degree of visibility, by reason of its definiteness and significance, is used as a rational basis for developing a scale of relative visibility. It represents a factor of safety of unity under ideal conditions for maximum efficacy of the human seeing-machine. Such a degree of visibility is far below that desirable for seeing in practice.

Seeing is an intricate complexity in which learning plays an important part from birth. Human seeing-machines have trained themselves to concentrate their attention upon the central visual field. The reflex which causes the eyes to turn and fixate an extra-foveal stimulus makes it difficult to obtain reliable visibility measurements by peripheral vision. This phase of seeing, nevertheless, is highly important in situations where human safety is involved and for low brightness-levels where rod vision may predominate. Peripheral vision is particularly sensitive and useful in distinguishing motion which, from the viewpoint of seeing, means change in the brightness pattern. In general, measurements obtained with the visibility meter are those which result from foveal vision under various degrees of adaptation.

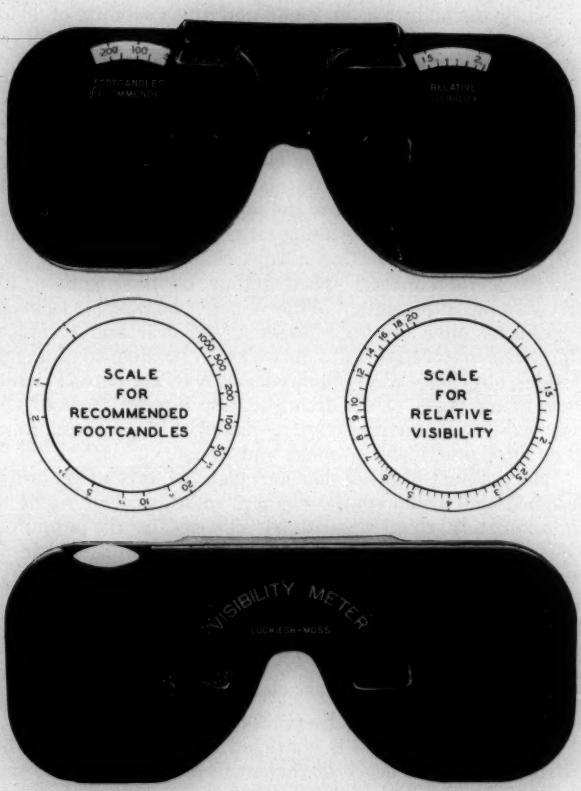
Visibility measurements usually involve only the phenomena which immediately follow the presentation of the stimulus. The appraisal of the relative visibility of traffic signals and signs may be cited as an example of the direct application of such data. However, the significance of threshold visibility measurements is greatly augmented by the establishment of correlations with certain psychophysiologic effects concomitant with critical seeing which are observable only after a more or less protracted period of visual effort. Increased tenseness,[7] changes in heart-rate,[8] and decreased convergence reserve[9] due to prolonged critical seeing are examples of such effects. We have shown that the magnitude of these effects for supra-liminal conditions corresponds to the visibility of the visual task when the latter is appraised by threshold measurements.[10]

THE LUCKIESH-MOSS VISIBILITY METER.

This instrument, illustrated in Fig. 2, consists essentially of two colorless photographic filters with precise circular gradients of density which may be rotated simultaneously in front of the eyes while looking at an object or while performing a visual task. The observer holds the instrument in approximately the same position that eyeglasses are worn, and with a finger of the right hand slowly turns a disk which rotates the circular gradients until the visual threshold or limit in the performance of the visual task is reached. The procedure is quite similar to that employed in operating a visual photometer. The contour of the instrument is such that it conforms to the shape of the forehead and bridge of the nose and, if both hands are used, the instrument may be held very steady against the forehead and the mechanical adjustments can be made with nicety. These characteristics are essential ones. Individual measurements of visibility may be made in a few seconds even by untrained observers. Obviously, the reliability of the appraisals of the relative visibility of various objects is proportional to the number of observations involved, assuming, of course, that obvious precautions are exercised.

The gradient filters not only reduce the apparent brightness of the visual field due to absorption, but also lower the contrast between the object of regard and its background due to the slightly diffusing characteristics of the photographic filters. The slight diffusion produces the effect of a "veiling" brightness over the field of view. The brightness of the retinal image may be varied by the gradient filters over a range which corresponds to 100 per cent. in visual acuity or the threshold size of the object of regard. Since the maximum range of the instrument corresponds to 20 times the threshold size, it is seen that the reduction in contrast is the more effective of the two factors in producing threshold conditions. Obviously, the state of retinal adaptation is a function of the density of the filters before the eyes. Since this variable is present in both the calibration and the use of the instrument, it does not introduce differential effects in the measurement of relative visibility. The range of our standard visibility meter is designed for everyday seeing conditions indoors in the daytime and under artificial lighting at night. This covers a very wide

FIG. 2.



The Luckiesh-Moss Visibility Meter. A new instrument for appraising the visibility of various objects and visual tasks and for specifying practicable levels of illumination for visual tasks upon a rational basis. The upper and lower illustrations show the side toward the observer's eyes and the reverse side, respectively. Between these are shown the scales of Recommended Footcandles (left) and of Relative Visibility (right) in detail.

range, but there are some cases where special gradients are desirable. We have studied these special cases such as visibility on highways at night and the visibility of light-sources, beacons, etc., but a discussion of them is beyond the scope of this paper.

A monocular instrument, known as the Bennett-Casella Visibility Meter,[11] provides a means for obtaining threshold conditions by interposing a series of slightly ground glasses in the line of vision. Hence the visual threshold is approached by discrete changes in the contrast between the object and its background. In addition to the shortcomings of a monocular instrument the inevitable ambiguity in the criterion of threshold visibility which follows abrupt changes in filters is considered a fundamental disadvantage of this method for precision measurements. While it might be possible to make a gradient filter of this character, the cost and technical difficulties involved seem to outweigh its possible advantages. Besides, our choice was influenced largely by the data available from many years of research partially directed toward the eventual development of a seeing-meter which appeared to us to be most practicable to make and to use.

The gradient filters of our binocular visibility meter are so designed that equal spatial changes in adjustment result in approximately equal logarithmic changes in the stimulus. Thus the mechanical operation of the instrument is similar, or nearly so, throughout the entire range of relative visibility. Actually, the slope of the density curve, plotted on semi-logarithmic coordinates, increases at the mid-region of the circular filter. This characteristic of the gradient involves a compromise between (1) maximum range in visibility and (2) an extended scale in the less dense region of the filter through which many work-world observations are made.

Since the visual field is restricted by the apertures of the instrument, the influence of the entire peripheral field is not included as a variable in the measurement of relative visibility. However, it has been shown that the brightness characteristics of the surrounding fields, excluding glaring light-sources, are not important when the area of the central field exceeds that subtended by an angle of approximately 30 degrees.[12] This was taken into account in designing the present instrument.

The field of view for each eye subtends horizontal and vertical angles of 30 and 24 degrees respectively. Hence the *critical* portion of the peripheral field is within the field of view. These conclusions pertaining to the distribution of brightnesses within the peripheral field are based upon considerations of threshold visibility and do not include the undesirable psychophysiologic effects which may be produced by improper lighting outside the central field.

The instrument is provided with two rational scales:

Relative Visibility . . . . . Scale range, 1 to 20
Footcandles Recommended . . Scale range, 1 to 1000

The scale termed "relative" visibility can be "absolute" visibility by definition. In fact it is as absolute as visibility can be. The scale of recommended footcandles is rational and absolute within itself, but the general level is necessarily an arbitrary compromise between the ideal and the practicable. The calibration technique is summarized in Fig. 1 and discussed in detail in sections of this paper which follow.

The instrument has extensive applications in research and in practice. Every case in which critical seeing is involved and every aspect of critical seeing may be studied with it. Throughout the range of seeing which includes the visual task, the visual sense, and light and lighting, this instrument is applicable. Studies of parts or of the whole can be made as desired. The great number and variety of uses preclude discussion of this aspect beyond that presented in other parts of this paper.

SCALE OF RELATIVE VISIBILITY.

A fundamental characteristic of the visibility-meter technique is that all observations involve threshold stimuli and therefore represent the *same degree of visibility*. Since differences in the visibility of various objects of regard may be compensated by altering the position of the gradient filters before the eyes, the relative visibility of an object may be expressed as a function of the density of the filters producing threshold conditions. Furthermore, the effectiveness of the filters in altering visibility is not a function of the distance, size, brightness, or color of the object of regard. It follows, *a priori*, that the determination of the relative visibility of

various objects, for the specific conditions under which they are viewed, is independent of the conditions under which the instrument is calibrated. This conclusion involves the assumption that the scale of relative visibility is based upon the characteristics of the visual sensory processes. The significance of the scale of relative visibility is augmented if the unit of visibility is a rational rather than an empirical one. Our unit is based upon the resolving power (or acuity) of average normal eyes under certain prescribed conditions for seeing.

Although relative visibility may be expressed in terms of any one of the four fundamental variables of the visual threshold, the factor of areal extent or size possesses certain inherent advantages for this purpose. Visual acuity has long been a means of appraising vision. Besides, such data are easily translated into relative distances at which the various objects are just visible. Hence the significance of such appraisals of visibility is obvious to the vast majority of non-technical persons whose safety and welfare depend upon seeing and who generally exercise control over conditions for seeing in practice. While the basis for calibrating the instrument is a comparatively simple one, there are numerous specific factors to be considered in order to evolve a scale which is not only rational, but also quantitatively related to the visual ability of the observer. As many factors in seeing are involved in the calibration, the value of the instrument is increased by an understanding of them.

Since the visual acuity of an observer is a function of the fixational distance of the stimulus,[13] this factor is specified in the technique of calibrating the visibility scale. In general, critical visual tasks are performed at distances within arm's reach, as in reading, or at much greater distances, as in driving an automobile. In ophthalmology, these cardinal distances are usually assumed to be about 14 inches and 20 feet, respectively. Therefore, the instrument might be calibrated, for formal completeness, for both near- and distant-vision. However, the ultimate purpose of the instrument is to provide a means for appraising the visibility of objects or tasks in terms of factors which are controllable in practice—and to accomplish this as simply as possible. For near-visual tasks, intensity of illumination is an important and universally

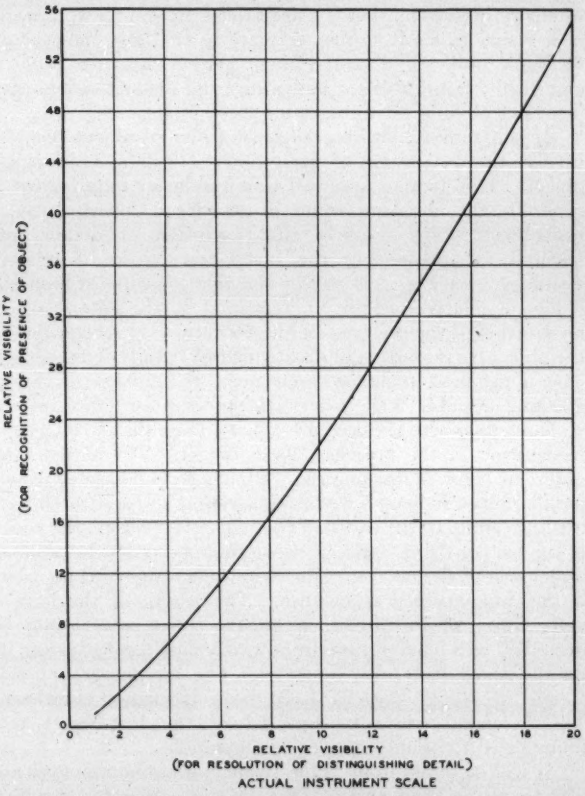
applicable factor for improving conditions for seeing. Since intensity of illumination is a desirable criterion for appraising near-visual tasks, it seems adequate to calibrate the scale of visibility only for distant-vision. Although either scale is empirically applicable for both near- and distant-vision, each possesses obvious advantages for specific purposes.

It is assumed that observations of distant objects usually involve the recognition of single objects or details of a single object. It is further assumed that the latter requirement is generally the more important in practice. Consequently, a test-object which requires the resolution of details was selected for calibrating the instrument scale of relative visibility. In Fig. 3 is shown the relationship between the values of relative visibility determined by the criteria of (1) recognition of the presence of an object and (2) recognition of the object by resolution of distinguishing details of the object. The instrument scale is based upon recognition of distinguishing details. Fig. 3 permits the transformation of the scale readings into the cruder form of seeing, that is, into mere recognition of the presence of an object. The actual test-object used for calibrating the visibility scale consisted of two parallel black bars on a white background and separated by a distance equal to the width of the bars. Therefore, the width of the bars or the distance between them becomes the nominal dimension of the object. This test-object was used for years in the foundational researches. The length of the bars is three times the width of each and the entire pattern may be described as a black square from which a strip constituting its middle third has been removed. The advantages of this simple geometric pattern have been discussed elsewhere.[6] The extended white background of the test-object was illuminated to an intensity of 10 footcandles.

It will be noted from Table I that normal human eyes are able to resolve, under favorable conditions, two objects which are separated approximately one minute in visual angle and viewed from a fixational distance of 20 feet. In this case, the visual acuity is unity.

Visual acuity of this order represents the average power of resolution of normal eyes. In cases of exceptional visual acuteness, a critical detail subtending an angle of 30 seconds

FIG. 3.



A chart for obtaining values of relative visibility based upon the criterion of recognition of the mere presence of an object (ordinates) from the actual instrument scale values (abscissae) which are based upon the criterion of resolution of critical details of the object.

may be recognized. Some of the more important factors which modify acuity are summarized in Fig. 1. The results of numerous researches indicate that the critical detail of a black object upon a white background, subtending an angle of one

TABLE I.*

Observer.	Value at Retina in Microns.	Angle in Seconds of Arc.
Lister	4.6	64
Hirschmann	3.6	50
Bergmann	3.8	52
Helmholtz	4.6	64
Uthoff	4.0	56
Cobb	4.6	64
		Average 58 seconds

* After F. H. Adler.

minute at a distance of 20 feet, and illuminated to an intensity of 10 footcandles, represents a test-object of threshold visibility for an observer possessing average normal vision. The specification of distance is introduced since visual acuity is a function of the stimulus distance. The visibility of an object of this description represents *unity* on our scale of relative visibility. In a technical sense, this specification of unit visibility is somewhat arbitrary, but in the world of seeing, it is rational, quite exact and even basic.

A visibility-scale value of "1" indicates that the standard test-object whose critical detail subtends a visual angle of one minute, is of border-line visibility when viewed through the corresponding region of the filter. Hence any object, under any conditions, which is barely visible when observed through this portion of the gradient filter is assigned a value of unity in visibility. A scale value of "2" means that the visibility is equivalent to that of our standard test-object when the critical detail of the latter subtends a visual angle of two minutes. Other cardinal scale-values were obtained by a similar method and have a similar significance. These values of relative visibility assume an arithmetic relationship between threshold visibility and the areal characteristics of the standard test-object. According to this visibility scale an object twice as large as another is twice as visible, provided other factors are constant. However, this does not mean that the visibility of the larger object is such that it serves the human seeing-machine in the ratio indicated. For example, let it be assumed that the visual rating of two subjects by means of the Snellen chart is 20/20 and 20/40, respectively. Thus the threshold size for the first subject is one-half of that for the second subject. Hence from physical considerations,

it might seem reasonable to suppose that the visual ability of one subject is twice as great as that of the other. This supposition is contrary to experience of eyesight specialists.[14]

The scale values of relative visibility may be interpreted directly as "factors of safety" since this term is based upon physical factors. For example, a factor of safety of 10 in bridge design means that the structure is capable of withstanding a load 10 times greater than the maximum expected load which formed the basis of the computations. Similarly, a scale value of relative visibility of 10 indicates that the human seeing-machine is capable of recognizing an object one-tenth as large as the standard test-object corresponding to this scale value. In this respect the factor of safety is 10. Since human efficiency, safety, and welfare depend so largely upon seeing clearly and quickly, even while some attention is given to other matters besides seeing, the technical value of quantitative factors of safety in seeing is obvious. Acceptable or adequate factors of safety for the tasks of the home, school, work-world, on the highway and in other situations where critical seeing is involved remain to be formulated by the science of seeing. The visibility-meter technique offers, for the first time, a practical and rational basis for determining these factors.

As seen in Fig. 2 the scale of relative visibility varies logarithmically from a minimum value of 1 to a maximum of 20 for positions of the circular gradients from 40 to 340 degrees, respectively. This range is adequate for most practical uses since few critical tasks of seeing involve visibilities as high as 20. The important exceptions to this generalization are situations which involve visible light-sources, such as luminous signs or signals. Obviously, a quantitative appraisal of visibility becomes less important as the latter increases far above the threshold. Visual tasks performed outdoors under daylight are usually in this category. For researches involving very low visibility the range of the low-density portion of the gradients has been specially extended for greater sensitivity.

In much of the world of seeing it is important to know the relative visibility of objects or of visual tasks under the conditions as found. Then the effect of varying any con-

trollable factor may be further studied. For example, it may be more convenient to determine the relative visibility of various objects under the existing conditions of illumination and then apply correction-factors than to produce an illumination of 10 footcandles upon the objects. For example, let it be assumed that the relative visibilities of two different objects, as read from the scale of the instrument are 1 and 2 when the intensities of illumination are 0.5 and 20 footcandles, respectively. It will be noted from the data presented in Table II that correction (multiplying) factors of 1.43 and

TABLE II.

*The Relationship Between Size and Brightness as Variables of the Visual Threshold.**

Footcandles on standard test-object	0.5	1	2	5	10	20	50	100	200
Relative size of standard test-object	1.43*	1.32	1.21	1.08	1.00	0.92	0.82	0.75	0.69*

* Extrapolated.

0.92 are required for these levels of illumination in order to obtain comparable values of relative visibility *under the same intensity of illumination*. By multiplying the scale readings, 1 and 2, by 1.43 and 0.92, respectively, the relative visibility values under 10 footcandles are found to be 1.43 and 1.84. It is assumed that the fixational distance is the same in both cases, although not necessarily equal to that used in the calibration of the instrument.

The rationality of the calibration of the instrument, as it pertains to certain photometric characteristics of the test-objects, was also tested by comparing the visibility measurements of our standard parallel-bar test-object corresponding to (1) black bars upon white backgrounds and (2) white bars upon black backgrounds. The two methods of calibrating resulted in nearly identical scales of relative visibility. Therefore, the scale of relative visibility may be assumed to be rational for both types and all degrees of contrast less than 100 per cent. For this purpose brightness-contrast can be defined as the ratio of the brightness-difference between object and background to the brightness of the brighter of the two. The contrast between black printed matter and white paper

approaches 100 per cent. Likewise a photographic negative of the printed matter may also approach this value.

A statistical appraisal of the variability of the visibility measurements obtained by the same observer at different times or by different observers is possible from the data obtained in calibrating the present visibility meter. These data involve a series of 5 observations of relative visibility obtained on each of 5 days for 8 different test-objects by 14 observers previously unfamiliar with the instrument. In summarizing these data, the arithmetic average of the 5 serial observations was regarded as a single measurement of relative visibility. The results indicated a mean variation in the measurements of relative visibility of approximately 10 per cent. when variations due to systematic differences among observers were *excluded*. It was also established that the variability of the measurements was practically independent of the physical characteristics of the object of regard for these conditions.

The significance of a mean variation of 10 per cent. in the threshold size of the object of regard, as measured at different times, may be appreciated by reference to the letters on the usual test-chart of the eyesight specialist. The letters of such a card increase in size by successive increments which vary from 20 to 100 per cent. Hence it follows that the mean variation of measurements of relative visibility obtained by the same observer at different times is less than the difference in the successive sizes of letters on the visual acuity test-chart. Since the gradations of the test-chart are considered adequate for the clinical appraisal of visual ability, it seems reasonable to conclude that the reproducibility of threshold visibility measurements is adequate for the appraisal of the relative visibility. Besides, there is no other promising method of eliminating the vagaries of the human seeing-machine which are responsible for the variation among the measurements.

The mean variation of a single measurement of relative visibility, *including* systematic differences among observers, was approximately 30 per cent. when the observations were made through the central region of the circular gradient. The magnitude of this factor depends upon the physical characteristics of the object of regard as well as upon the character-

istics of the observers. In general, it may be concluded from the data available that the variability of results obtained by different observers of normal or near normal vision is several times as great as the variability of results obtained by given observers at different times. However, an instrument for measuring visibility would scarcely be worthwhile or sensitive enough if it *did not* yield different results for observers differing in visual ability.

It is obvious that the subject who is handicapped by visual deficiencies requires more favorable conditions for seeing than does the subject possessing normal vision. Furthermore, it is conceivable that one subject may require more favorable conditions for seeing than does another for purely physiologic or psychologic reasons. In the specification of conditions for seeing, it would seem immaterial whether the visual limitations were imposed by morphologic, physiologic or psychologic factors. Hence, the values of relative visibility obtained with this instrument are *inherently proper* for the subject who appraises the conditions for seeing.

In certain situations, the recognition of the mere presence (or absence) of an object may be a more important criterion of visibility than the resolution of details of the object. Usually such situations involve distant-vision, as on the highways, although there are exceptions involving near-vision such as inspection tasks in the work-world. If the visibility of an object is gradually lowered, the thresholds of resolution, form and presence are successively reached. In the last stage of visibility, test-objects in the form of squares, triangles, circles or ovals appear merely as blurred patches which are more or less circular in shape. Hence a circular test-object is a rational one for developing a scale of relative visibility which is based upon the recognition of the mere presence of an object.

The data of McCallie [15] and of others,[16] as summarized by Southall, indicate that a black circular object (dot) upon a white background is of threshold visibility for average normal eyes when it subtends a visual angle of approximately one minute. This value is in close agreement with the result we obtained in calibrating the visibility meter by the criterion of the recognition of the presence of the test-object. Hence an object of this description is assigned a value of unity on our

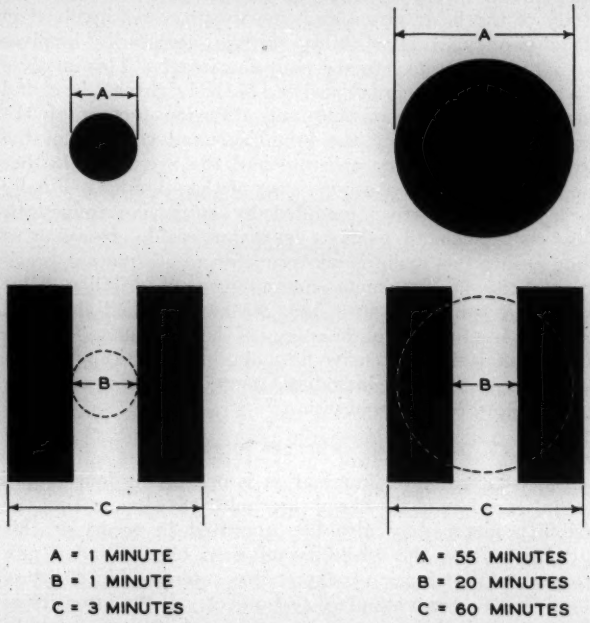
scale of relative visibility pertaining to the recognition of the presence of an object. Thus the value "1" on the scale of relative visibility corresponds, on both scales, to the same density of the circular gradient. However, from this point the results of calibrations for the two types of visibility depart from each other. The relationship between the two scales of relative visibility is presented in Fig. 3.

A discussion of these two different criteria—recognition of an *object* and of merely the *presence* of an object, respectively—leads quickly into an uncharted realm. The scanty data heretofore available, comparing these two types of visibility, deal only with high-contrast test-objects and, therefore, with the minimum sizes that are visible under highly favorable circumstances. These threshold sizes are of the order of one minute visual angle and in these cases there seems to be little difference between the results by the two types of criteria. Retinal images of such threshold sizes are comparable with the area occupied by a single element of the retinal mosaic.[17] Doubtless this anatomical fact has much to do with the preceding experimental fact.

The two diagrams in Fig. 4 show the relative threshold sizes of the two types of test-objects for each of the two extreme conditions, respectively; that is, at the two ends of the curve in Fig. 3. It is not practicable to draw the left-hand and right-hand diagrams in Fig. 4 to the same scale; therefore, the scale of the left-hand diagram has been made 20 times that of the right-hand one. The actual test-objects were black against a white background. Practically the same results are obtained if they are white on a black background. Incidentally, the observers experienced much more difficulty in making threshold visibility measurements by the criteria which involved merely the presence of an object than they experienced when the criteria involved resolution of distinguishing details or the recognition of an object.

The actual objects were of high contrast, but when seen through the visibility meter and, therefore, as retinal stimuli, the contrast was not constant. Owing to the slight diffusion of the gradient filters the contrast varied from nearly 100 per cent. for the smallest threshold sizes to a low contrast for the largest sizes. This characteristic of the gradients com-

FIG. 4.



Threshold sizes of two test-objects representing the two types of visibility criteria for the two ends of the scale of the visibility meter. The data correspond to those at the two ends of the curve in Fig. 3.

bined with their variation in transmission-factor contributes greatly to the range and satisfactoriness of the visibility meter. Naturally this must be taken into account in discussing the theory of visibility based upon measurements with this instrument.

The visibility of an object is intimately associated with the total radiant energy of the stimulus at the retina; that is, the total energy over the area of the retinal image. When the object is darker than the background we have termed this "negative energy" because it represents the energy subtracted from the visual field. When the object is brighter than the

background energy is added to the background. A critical study of the basic visual and sensory processes involved in these two types of visibility criteria necessarily involves energy of the actual retinal stimulus or image. This involves the area of the object of regard and of the brightness-level and contrast. The transmission and diffusion factors of the gradient filters modify the brightness and contrast of the object. The pupillary aperture and the eye-media further modify the physical characteristics of the stimulus. Finally these may be further modified by subjective influences. Obviously, such an analysis leads far afield. However, a glimpse into this realm is necessary for an adequate appreciation of the significance and measurement of visibility. But perhaps it suffices to state that we have studied threshold visibility as a function of the energy of the stimulus. Through such an analysis of extensive data obtained with the parallel-bar test-object we have introduced new conceptions pertaining to the theory of threshold vision.[17]

SCALE OF RECOMMENDED FOOTCANDLES.

Since intensity of illumination is one of the four fundamental variables of the visual threshold, it is obvious that the visibility meter may also be calibrated in terms of this variable. Thus the visibility of any object under any intensity of illumination is expressible in terms of intensity of illumination upon a standard test-object. If the intensity of illumination incident upon the various objects of regard is constant, and equal to that under which the instrument was calibrated, the scale values represent the intensities of illumination required upon such objects in order to produce the same degree of visibility as that of the standard test-object. Furthermore, if the intensity of illumination upon the standard test-object is considered satisfactory for seeing the object, the scale values are likewise significant as *recommended* intensities of illumination. Therefore, it is now possible to specify intensities of illumination for various visual tasks upon a rational basis. Inasmuch as light is a very controllable factor in our complex civilization, in which the specification of light has been largely a matter of guesswork and lighting has been little more than a crude art, the usefulness of this

visibility meter is greatly extended by providing a rational scale of Recommended Footcandles.

The technique of calibrating the scale of recommended footcandles involves the following steps: (1) a determination of the relationship between density of the gradient filter and intensity of illumination corresponding to threshold visibility of the standard test-object; (2) the assumption that a given intensity of illumination is "acceptable" for seeing an object of a certain description; and (3) a transition from *threshold* to *recommended* intensities of illumination. The technical details of these phases of the calibration process are briefly discussed in this order.

Critical visual tasks are usually performed at distances within arm's reach. This distance is about 14 inches for adults and was used in calibrating the scale of recommended footcandles. It is obvious that the tasks performed at distances greater than 14 inches are visually more difficult than they would be at shorter distances. Hence correspondingly higher intensities of recommended footcandles will be obtained. The distance of observation at which this visibility meter is used in practice is independent of that used in calibrating it. However, the observation distance at which it is used in practice for determining the recommended footcandles for a given visual task should be sensibly the same as the distance from the eyes to the task as it is commonly performed. Those not used to thinking in terms of "visual size" are reminded that the distance from object is just as much a factor as the actual physical size of the object.

It seems reasonable to assume that reading is the most universal of the near-vision tasks, and since it is a critical, uniform, describable and controllable visual task, it possesses distinct advantages as a standard of comparison for calibrating the scale of recommended footcandles. Many of our researches have involved reading as the visual task and, therefore, much pertinent knowledge and experience is available as a sound basis upon which to build. In addition, the fact that the printed words lie in a single plane minimizes the influence of possible variables in the quality of the lighting. Since the visual threshold is generally approached by gradually increasing the density of the filters before the eyes while reading,

it is possible that continuity in the text may be a factor in the determination of threshold conditions. Ambiguity due to this cause was minimized by inserting irrelevant words (numerals) throughout the text. The reading matter consisted of several paragraphs printed in black 8-point Bodoni book-type on an excellent grade of non-glossy white paper. This particular size and style of type-face was selected because of its prominence in typography.[18] It is further assumed that the visual pattern presented by the individual letters (or words) is comparable in visual difficulty to that of many tasks of the work-world although, of course, many tasks are far more difficult.

The relationship between density of the gradient filter and intensity of illumination was determined for threshold conditions of seeing for various levels of illumination between 0.1 and 100 footcandles. Since the filter density, for threshold conditions, is proportional to the visibility of the object of regard, a correlation between visibility (or ease of seeing) and intensity of illumination may be derived from these data. The rationality of the resultant scale relating these factors, for various types of visual tasks, has also been investigated and established by methods which are discussed later.

It has been definitely established that intensities of illumination of at least 100 footcandles are desirable for such a visual task as reading ordinary black print upon white paper. This conclusion is firmly based upon the results of extensive researches in the physiologic effects of seeing and is in harmony with philosophical considerations and experience. Some of the more important criteria for determining ideal intensities of illumination for critical visual tasks are summarized as follows:

Momentary Criteria of Visual Efficiency
: Visual acuity; contrast sensibility; speed of retinal impression.

Integrative Phenomena
: Nervous muscular tension; ocular muscle fatigue; heart-rate decrement.

Philosophical Considerations
: Evolution under daylight intensities of illumination.

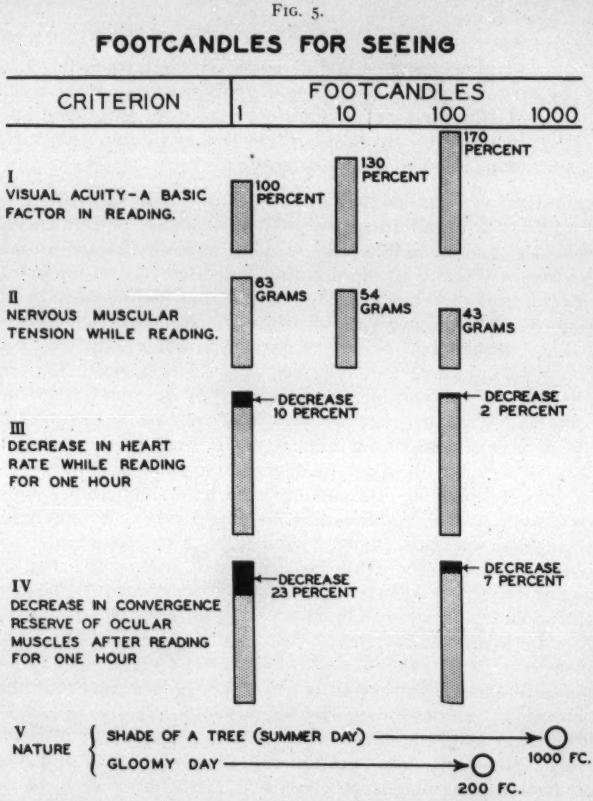
Experience

Actual selection of desirable levels of illumination by many subjects upon a basis of what seemed to be comfortable seeing; general tendency to increase levels of illumination as light-sources were improved; reluctance to use lower intensities of illumination after being used to higher intensities.

Quantitative data related to these criteria are presented in Fig. 5. The standardization of 10 footcandles for calibrating the scale of recommended footcandles [19] involved a compromise between the levels of illumination indicated by the foregoing criteria and the economic and engineering factors of lighting practice. The conservativeness of this standard of "ease of seeing" should be obvious to anyone familiar with the new concepts and knowledge of seeing. It is emphasized that the authors do not consider an intensity of illumination of 10 footcandles as adequate for *prolonged* reading, or any other visual task of comparable difficulty. It was adopted because it is conservative beyond question and argument.

The method of converting threshold footcandles into recommended footcandles may be explained by an example. If an object X is of higher visibility than the standard test-object S, under the same intensity of illumination, then X would require a denser filter than S in order to produce threshold conditions. Obviously this differential in density may be expressed in terms of footcandles upon the standard test-object S by reference to the data determined in the first phase of the calibration process. Let it be assumed that the threshold densities for the two objects correspond to threshold intensities of illumination of 20 and 10 footcandles, respectively. Since the differential in visibility between X and S is of such magnitude as to require a factor of 2 in level of illumination in order to produce an equality in visibility, it follows that 5 footcandles should be recommended for X if 10 footcandles is considered adequate for S. This relationship may be expressed by the formula, $R = 10/T \times 10$, where R and T are the recommended and threshold footcandles, respectively; and the constant 10 is the level of illumination upon the standard test-object used in the calibration of the scale of recommended footcandles.

FIG. 5.



Fundamental criteria for appraising the relationship between intensity of illumination and ease of seeing.

Since the determinations of recommended footcandles are based upon threshold observations, it follows that the values obtained depend upon the criterion of threshold visibility involved in the measurements. The instrument is calibrated upon a basis of resolution and hence the scale values are

applicable to visual tasks involving this type of vision. Obviously, the intensity of illumination required for resolution of details is higher than that required for merely recognizing the presence of the object. If the latter criterion is used, the level of recommended footcandles will represent a condition for performing the task in practice which is inferior to that selected as a conservative standard of ease of seeing.

The rationality of this procedure for relating threshold and recommended intensities of illumination has been investigated by direct methods. For this purpose four tasks were selected differing greatly in size of the object of regard, in brightness contrast, in color contrast, and in the interpretation of the visual stimulus. These tasks were diffusely illuminated to an intensity of 10 footcandles, the instrument adjusted for threshold seeing, and the value of recommended intensity of illumination for each task was read from the scale. The validity of these recommended footcandle values was then checked by illuminating the individual objects to their respective values of recommended footcandles and again adjusting the instrument for threshold seeing. If the method of relating threshold and recommended footcandles is a rational one, it is obvious that a scale value of 10 footcandles should be obtained for *all* tasks. It will be noted in Table III that this requirement is fulfilled in fact, within the limits of accuracy of the instrument. The differentials between Items *A* and *D* of Table III are therefore considered as errors of measurement, primarily, rather than as criteria of irrationality in the calibration of the instrument. Furthermore, it will be noted that they have little significance when appraised upon a basis of the quantitative effectiveness of intensity of illumination as a factor in seeing.

In view of the relatively few observations involved in the summary presented in Table III and, particularly, in view of the basic fact that footcandles must be altered in geometric ratio in order to alter visibility or visual effectiveness in arithmetic ratio, the per cent. error in any case in Table III is insignificant. With these facts in mind, these data appear to establish the rationality of the scale of recommended footcandles, determined from a single type of test-object (8-point Bodoni type), for objects of diverse physical characteristics.

TABLE III.

These Data Present the Specification of Recommended Footcandles for Four Diverse Visual Tasks and Evidence of the Rationality of the Values Obtained.

Order of Procedure in Checking Rationality of Transition from Threshold to Recommended Footcandles.	Visual Tasks.			
	Use of Steel Scale, 1/64 Inch Divisions.	Reading Newspaper Stock Quotations.	Dark Gray Print Upon Lighter Gray Background.	Typing on Dark Blue Paper.
Measurement				
A. Footcandles on visual tasks. (Standard for calibration.)	10	10	10	10
B. Footcandles recommended for particular task by instrument	160	84	171	88
Check				
C. Footcandles on tasks determined in Step *B*	160	84	171	88
D. Footcandles read from scale. These values should correspond to those of Step *A*	10	11.5	13.4	11
Percent error (D/A)	0	15	34	10

Hence the intensities of illumination for various tasks correspond directly to the scale values when the measurements are made under an intensity of illumination of 10 footcandles. When the measurements are made under an actual level of illumination, A, which is different from that (10 footcandles) for which the instrument is calibrated, a correction-factor must be applied to the scale values. It is obvious that the actual intensity of illumination, A, under which any observation is made, is equivalent to T in the calibration formula, $R = 10/T \times 10$, already discussed. Therefore, the scale values of recommended footcandles must be multiplied by the factor $A/10$ or $0.1A$. For example, if an object is observed, or a task is performed, under an intensity of illumination of 1 footcandle and a scale value of 50 footcandles is obtained, the recommended footcandles will be one tenth of 50 footcandles or 5 footcandles. If the actual intensity of illumination under which measurements are made in another case is 20 footcandles and a scale value of 50 footcandles is obtained, the recommended footcandles will be twice 50 footcandles or 100 footcandles.

It will be recalled that the standard of "ease of seeing" selected for calibrating the scale of recommended footcandles was that corresponding to 8-point Bodoni type printed in black upon white paper and illuminated to an intensity of 10 footcandles. We consider this degree of ease of seeing far from desirable but certainly it is the minimum for normal adult eyes under otherwise favorable conditions for seeing. In the case of young, old or defective eyes, or for less favorable conditions for seeing, a higher standard should be the minimum. The scale values of the present instrument are easily transferred to other standards of ease of seeing. For example, 10 footcandles upon 12-point type is a more satisfactory standard than the same intensity of illumination upon 8-point type. If the latter standard is adopted for normal adult eyes, it may be assumed that the 12-point standard, or even a higher one, is more appropriate for old or defective eyes. Other standards of ease of seeing may be specified upon a basis of intensity of illumination upon a given object of regard.

Correction-factors for various standards of ease of seeing are presented in Table IV. These factors are derived from

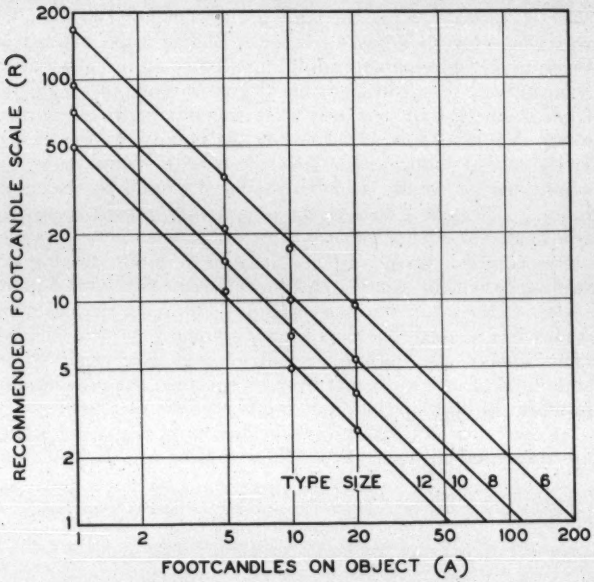
TABLE IV.

Correction-factors for the Scale of Recommended Footcandles Corresponding to Various Standards of Ease of Seeing.

	Type Size in Points			
Standard of Seeing 10 Footcandles on	6	8	10	12
Multiplying Factors for Footcandle Scale . .	0.6	1.0	1.3	2.0
	Footcandles upon Test-Object			
Standard of Seeing 8-Point Book Type	5	10	20	50
Multiplying Factors for Footcandle Scale . .	0.25	1.0	4	25

the experimental data of Fig. 6 relating the actual intensity of illumination upon the object of regard and the corresponding scale value of recommended footcandles. It will be noted from Fig. 6 that similar relationships between these variables are obtained for the various sizes of type. Therefore, the scale values of recommended footcandles may be corrected for other standards of seeing upon a rational basis.

FIG. 6.



Basic data for determining recommended footcandles for various standards of ease of seeing

SUMMARY.

The complex chain of factors and events resulting in visibility is discussed, and its significance and measurement in seeing are emphasized. Researches in seeing,[20] as contrasted with more restricted researches in vision, have established certain facts, laws and relationships connecting barely seeing with easy seeing. This makes possible the interpretation of threshold measurements into more valuable and practicable recommendations of footcandles for easy seeing and also factors of safety in visibility. Such interpretation is essential if threshold measurements, which are readily accomplished by human seeing-machines, are to be elevated from mere theoretical value to real practical value in everyday seeing.

As a consequence a Visibility Meter has been developed which possesses rational scales of visibility and recommended footcandles. Details of its theory, foundation, calibration, and practical significance and use are presented. The specific instrument described is designed for wide use indoors in the daytime and everywhere artificial lighting is used at night. Highly visible objects outdoors in the daytime and light-sources such as signals and beacons at night are beyond its normal range, but special filters have been made to cover these when necessary.

The rationality of the scales is an achievement of great practical importance. In the actual calibrations, as in the previous foundational researches, subjects having average normal vision were used. This guarantees rationality in every respect. Unity on the visibility scale represents the point at which two objects, separated by one minute visual angle and illuminated by ten footcandles, can be barely distinguished. This scale deals with actual recognition of objects. When the cruder form of visibility—mere recognition of the presence of an object—is sufficient, the present visibility-scale values can be multiplied by certain constants having values from 1 to 3.

The value of 10 on the scale of recommended footcandles is the point at which 8-point Bodoni type (printed with black ink on white non-glossy paper) is barely readable under ten footcandles. The entire scale is rational inasmuch as it is based upon the relationship of footcandles and visibility established by extensive researches during recent years. The general level of these recommended footcandles is a compromise of many aspects of lighting. Factors are given for transforming these values to various other bases.

In seeing, human beings are interested in the visual task, in the visual sense, and in light and lighting. This Visibility Meter is applicable to a vast variety of inquiry throughout these three realms. Its precision is adequate for research and its portability is satisfactory for manifold practical applications. Space does not permit listing even major applications pertaining to tasks, vision, light and lighting. Such a listing is scarcely necessary when it may be said that this instrument is applicable to all cases and aspects of critical seeing Owing to the importance of measurements in the

growth of knowledge, development and practice along any line, it may be safely predicted that this Visibility Meter can do much toward unraveling the complexity of seeing and in eliminating present guesswork, inconsistency, and ambiguity in this most universal and important activity of human beings.

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STEADY FORCED VIBRATIONS OF SINGLE MASS SYSTEMS WITH SYMMETRICAL AS WELL AS UNSYMMETRICAL NON-LINEAR RESTORING ELEMENTS.

BY

L. S. JACOBSEN[1] and H. J. JESPERSEN.[2]

Part of the theory and all the experiments reported on in this paper were condensed into a four-page discussion or addendum to Professor Den Hartog's original paper on "Amplitudes of Non-Harmonic Vibrations" at the time of its presentation at the June Meeting of the A.S.M.E. in Chicago, 1933. Upon the instigation of Professor Den Hartog and the Editor of the JOURNAL OF THE FRANKLIN INSTITUTE, the original discussion has been brought into the present form.

INTRODUCTION.

In connection with the possibility of designing buildings with a so-called flexible first story to resist seismic disturbances, it was pointed out by one of the writers several years ago that the danger of resonance with the lowest mode of vibration of a building might possibly be reduced somewhat by providing the first story with a restoring element of non-linear characteristic. It was soon realized, however, that a knowledge of the response of a single mass system with non-linear spring characteristic to a simple harmonic ground motion had to be obtained before quantitative application to multiple mass systems could be made, and that a solution of even the single mass system involved considerable difficulty. The admirable paper by Messrs. Den Hartog and Mikina in the *A.S.M.E. Transactions*, Vol. 54, indicated clearly that an approximate solution had to be resorted to if the results were to be at all manageable by designers.

The general, idealized problem is therefore to find particular solutions of the differential equation

$$m\ddot{x} + f(x - x_0) = 0, \qquad (1)$$

in which x_0 is a known function of time. In this paper it

[1] Associate Professor, Stanford University, Calif.

[2] M.E., Shell Chemical Co., Pittsburgh, Calif.

will be assumed that no frictional effects are present; moreover, that x_0, the instantaneous ground displacement, is given by

$$x_0 = c \sin \omega t. \tag{2}$$

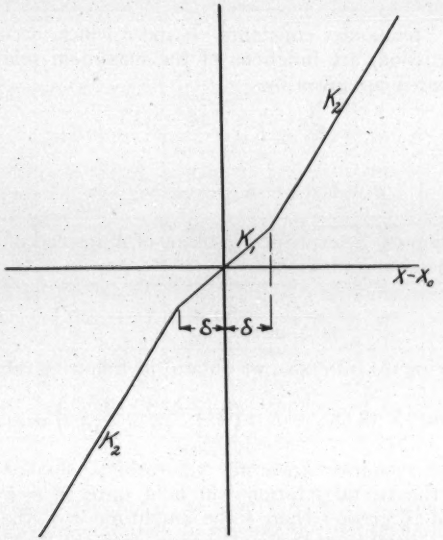
Until Professor Den Hartog's paper on "The Amplitudes of Non-Harmonic Vibrations" had appeared in the October, 1933, issue of THE JOURNAL OF THE FRANKLIN INSTITUTE, the authors of this communication had employed a rather cumbersome analysis which, from the point of view of elegance, is much inferior to the one that is obtained by following Den Hartog's line of reasoning. Since, however, the authors' analysis gives the same results to a first approximation as the single term solution obtained from Den Hartog's analysis, it may be of interest to indicate briefly the solution for a system with a particular type of restoring force.

THEORY.

In the following approximate analysis of the steady forced vibration of a single mass system with two springs, one of which is inoperative until the displacement relative to the ground reaches a certain value δ, the discontinuous spring force curve, Fig. 1, has been approximated by a cubic parabola involving two parameters, α and β, which are functions of the maximum displacement X relative to the ground. The functional relationship of α, β, δ, and X rests on the criterion of equal potential energy in the actual spring system and in the fictitious, but approximately equivalent system, described by the cubic parabola. This criterion alone does not fix the values of α and β, as it is still possible to have an infinite number of ratios of the two parameters. A second criterion is necessary in order to determine α and β uniquely, and this criterion, which is more arbitrary than the first, consists in demanding that the maximum actual spring force, $K_1\delta + K_2(X - \delta)$, shall equal the maximum fictitious spring force, $\alpha X + \beta X^3$. The experimental evidence is in better agreement with this criterion than with, for instance, the one of equal potential energies of the two systems at δ as well as at X.

The differential equation of motion of the system may

FIG. 1.



Symmetrical restoring force curve used in equations (6), (6a), (7), and (7a).

then be written

$$m\ddot{x} + \alpha(x - x_0) + \beta(x - x_0)^3 = 0, \qquad (1a)$$

in which x and x_0 are the absolute displacements of the mass and of the ground. If it be understood that the ground motion is given by (2), and if the resulting motion be assumed to be simple harmonic, so that for the steady state

$$x = a \sin \omega t = (X \pm c) \sin \omega t \qquad (3)$$

the following equation results for the frequency ω:

$$\omega^2 = \frac{\beta}{ma}\left[a^3 \mp 3ca^2 + \left(\frac{\alpha}{\beta} + 3c^2\right) a \mp \left(\frac{\alpha}{\beta} c + c^3\right)\right]. \qquad (4)$$

The double signs refer to "in phase" and "180 degrees out of

phase" conditions, the upper one corresponding to "in phase" motion.

The "temporary constants" α and β which, according to our discussion, are functions of the maximum relative displacement X are given by:

$$\alpha = (K_2 - K_1)\frac{\delta(2\delta - 3X)}{X^2} + K_2,$$
$$\beta = 2(K_2 - K_1)\frac{\delta(X - \delta)}{X^4}. \tag{5}$$

If equation (4) be expressed in terms of X instead of in terms of a, the equation becomes

$$\omega^2 = \frac{X}{m(X \pm c)}[\alpha + \beta(X^2 - 2c^2)]. \tag{4a}$$

Introducing (5) into (4a), we obtain the following relation:

$$(K_2 - m\omega^2)X = (K_2 - K_1)\left(\delta + \frac{4c^2\delta}{X^2} - \frac{4c^2\delta^2}{X^3}\right) \pm m\omega^2 c. \tag{6}$$

Since the system is linear for values of X smaller than δ, Fig. 1, the trivial solution will hold until $X = \delta$. If for values of X greater than δ the amplitude c of the ground motion is small compared to X, as is often the case, we may neglect the terms involving X^{-2} and X^{-3} and write (6) as follows:

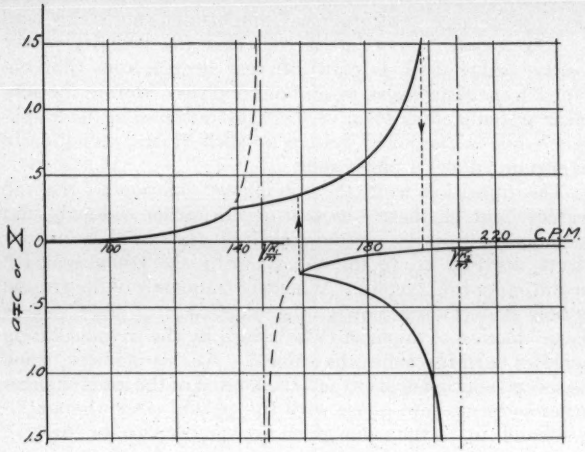
$$X = \frac{(K_2 - K_1)\delta \pm m\omega^2 c}{K_2 - m\omega^2}. \tag{6a}$$

The absolute motion is therefore given by:

$$a \sin \omega t = (X \pm c) \sin \omega t = \frac{(K_2 - K_1)\delta \pm K_2 c}{K_2 - m\omega^2} \sin \omega t. \tag{6b}$$

Figure 2 shows "resonance curves" for $c = .04$ inch; the maximum relative displacement X has been plotted for the system: $K_1 = \frac{1}{2}K_2 = 15.83$ pounds per inch; $m = .0664$ lbs./386, and $\delta = .25$ inch. When the frequency ω of the ground motion is increased slowly so that the relative displacement $x - x_0$ becomes greater than δ, the second spring begins to act and thereby destroys the possibility of resonance

FIG. 2.



Relative displacement of mass with respect to ground of which the steady motion is described by equation (6a).

$K_1 = \frac{1}{2}$, $K_2 = 15.83$ lbs./in.; $m = .0664$ lbs./386; $\delta = .25$ in.; $c = .04$ in.

amplitudes in the region of $\omega = \sqrt{(K_1/m)}$. When ω is increased further, equation (6) governs the motion and two sets of roots, each with four values of X are obtained, each set corresponding to the "in phase" or to the "out of phase" conditions. The two largest of these values of X are also given by equation (6a), and these approach plus or minus infinity as ω approaches $\sqrt{K_2/m}$. It would therefore seem that the effect of the second set of springs is merely to shift the "resonance frequency" from $\sqrt{K_1/m}$ to $\sqrt{K_2/m}$, incidentally calling for larger amplitudes than those corresponding to a simple system (dotted curves). We may perhaps, on account of the approximative nature of the analysis, dismiss the six smaller values of X from equation (6) as having no physical significance, but we cannot ignore the fact that equation (6a) has two distinct values of X, something entirely different from the characteristics of a simple system in which the relative amplitude X is a single valued function of ω. In the experiments the large negative value of

X was never observed; in other words, the experiments tend to show that a large "out of phase" amplitude is not stable, and for this reason it may be inferred that the stability of the positive value of X is of a different degree from that obtained in the simple system, an "out of phase" shift in the nonlinear system calling for a very drastic decrease in the amplitude, while in the linear system no such drastic reduction in amplitude follows a phase shift.

The transition from the "in phase" motion to the 180 degrees "out of phase" motion occurs rather suddenly and the amplitude drops, accompanied by the usual transitory effects, see Fig. 11, to the one given by the trivial solution for the one spring system. When the frequency of the ground motion is decreased slowly, the amplitude of the "out of phase" motion of the mass as described by the trivial solution increases until it reaches the value δ. At this moment, when the second spring begins to act, the motion of the mass changes rather suddenly into phase with the ground while the amplitude builds up to the value given by equations (6) or (6a).

STABILITY OF MOTION.

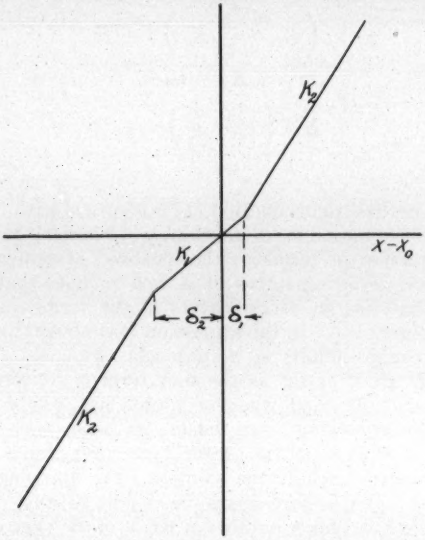
The question of stability has not been given analytical consideration for the practical case when the coefficients α and β are functions of the maximum relative amplitude, and it is questionable if such analysis will give quantitative results. When α and β are constant, the paper by E. V. Appleton of the Cavendish Laboratory[3] shows that the instability regions occurring in a vibration galvanometer in which the restoring moment is given by a cubic parabola are qualitatively in agreement with experimentally observed phenomena.

UNSYMMETRICAL RESTORING FORCE.

If the origin of the restoring force curve, Fig. 1, be translated along the first straight part of the curve, a type of unsymmetrical restoring force results, Fig. 3. The effect of such a condition on the motion of the mass will be to render the wave form unsymmetrical, and this, in turn, will complicate the approximate analysis much. The only obvious re-

[3] *Philosophical Magazine*, 47, p. 609.

FIG. 3.



Unsymmetrical restoring force curve used in equations (18), (19), (30), (31), (33), and (35).

lation that can be put down is that the energy of the system is wholly potential at the time of reversals, so that the ratio of the maximum displacements to the right and to the left can be easily computed by integrations of the spring force curves.

APPLICATION OF DEN HARTOG'S METHOD.

Symmetrical Case.

The aforementioned paper by Professor Den Hartog is so clear that the application of his method to the solution of equation (1) involves only obvious steps. When the restoring force is of the type shown in Fig. 1, a graphical solution is unnecessary, and the results become:

Single term solution:

$$a = \frac{(K_2 - K_1)\delta \pm K_2 c}{K_2 - m\omega^2}. \qquad (7)$$

Two term solution:

$$a_1 = \frac{\left(K_2 - \frac{2}{\sqrt{3}} K_1\right)\delta \pm K_2 c}{K_2 - m\omega^2}, \qquad a_3 = \frac{K_1\delta\left(1 - \frac{2}{\sqrt{3}}\right)}{K_2 - 9m\omega^2}. \tag{7a}$$

It will be noticed that equation (7) is identical with (6b).

Before we proceed to extend the use of Fourier's Series with a finite number of terms to the idealized system with unsymmetrical restoring forces, it is well to note that in this type of analysis the denominators of the terms vanish for various values of ω. In the expression (7a) above, the second term approaches infinity as ω approaches $\frac{1}{3}\sqrt{K_2/m}$. There is little doubt that, as far as the first term is concerned, the infinite value obtained when ω approaches $\sqrt{K_2/m}$ has its physical interpretation, but when, as above, a_3 becomes infinite at $\omega = \frac{1}{3}\sqrt{K_2/m}$, the physical interpretation is lacking, and we have to exclude the solution from applying in this region of ω. It therefore seems reasonable to state that in a Fourier Series of a finite number of terms of the type given by equations (7a), there will be regions of ω for which the solution has no physical interpretation.

Unsymmetrical Case.

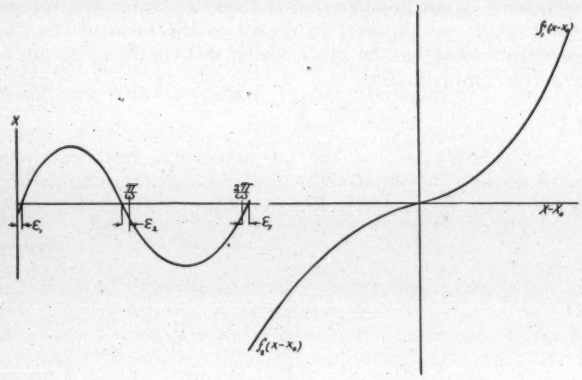
The paper by Den Hartog does not consider unsymmetrical restoring forces, but such may readily be dealt with by using sine as well as cosine terms in the general expression for a Fourier Series. We therefore assume that the motion shown in Fig. 4a may be represented by

$$x = \Sigma a_i \sin i\omega t + \Sigma b_i \cos i\omega t. \tag{8}$$

Moreover, it may be assumed that the unsymmetrical restoring force, Fig. 4b, expressed as a function of time is given by

$$f(x - x_0) = \Sigma A_i \sin i\omega t + \Sigma B_i \cos i\omega t. \tag{9}$$

FIG. 4a. FIG. 4b.



(4a) General time-displacement curve represented by equation (8).
(4b) General unsymmetrical restoring force curve used in equations (12), (21), and (27), also in Figs. 6a, 6b, and 7.

Introducing these expressions into equation (1), we find that the coefficients must have the following relations:

$$A_i = m\omega^2 i^2 a_i, \qquad B_i = m\omega^2 i^2 b_i. \tag{10}$$

Upon substitution of (10) into (9), we have:

$$f(x - x_0) = m\omega^2 \Sigma a_i i^2 \sin i\omega t + m\omega^2 \Sigma b_i i^2 \cos i\omega t. \tag{11}$$

Actual determination of the coefficients must then be governed by the following conditions:

$$\left.\begin{aligned} &\text{When } t \text{ is between } \epsilon_1 \text{ and } \frac{\pi}{\omega} - \epsilon_2, \\ &\qquad f(x - x_0) = f_1(x - x_0). \\ &\text{When } t \text{ is between } \frac{\pi}{\omega} - \epsilon_2 \text{ and } \frac{2\pi}{\omega} + \epsilon_1, \\ &\qquad f(x - x_0) = f_2(x - x_0). \end{aligned}\right\} \tag{12}$$

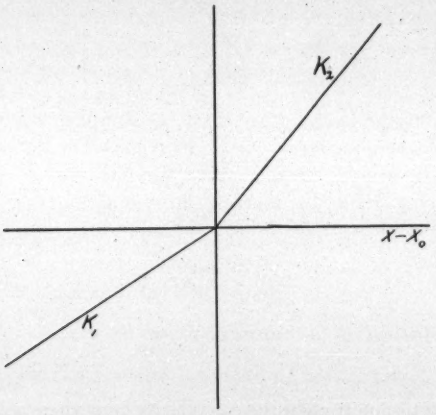
In the general case, the functions f_1 and f_2 are not readily

expressed analytically and an actual determination of the coefficients is not possible, but if these functions are not too complicated, assumptions in regard to the form of the displacement curve can be made which will enable us to arrive at a first approximation.

Special Case I.

Let us assume that the unsymmetrical restoring force curve consists of two straight lines through the origin, their slopes being K_2 and K_1, Fig. 5. Moreover, let K_2 correspond to f_1, the steeper curve, and let K_1 correspond to f_2, the curve with smaller slopes. It is then obvious that the mass will spend shorter time in the region governed by K_2 than in the one governed by K_1; the maximum displacement in the K_2 region will be shorter than the one in the K_1 region; finally, the maxima will occur when the times are $\pi/2\omega$ and $3\pi/2\omega$, etc. This means that ϵ_1 is equal to ϵ_2.

FIG. 5.



Unsymmetrical restoring force curve used in equations (14) and (28).

A single term trigonometric representation is therefore not

possible in this case. If we take two terms, we select, in view of the previous discussion,

$$x = a_1 \sin \omega t - b_4 \cos 4\omega t \tag{13}$$

as being the two terms that give promise of fitting the case reasonably well. Equation (11) then becomes:

$$\left.\begin{matrix} K_2(x - x_0) \\ K_1(-x + x_0) \end{matrix}\right\} = m\omega^2 a_1 \sin \omega t - 16m\omega^2 b_4 \cos 4\omega t.$$

When $t = \frac{\pi}{2\omega}$, we have the maximum displacement into the K_2 region:

$$K_2(x - x_0) = K_2(a_1 - b_4 - c) = m\omega^2 a_1 - 16m\omega^2 b_4.$$

When $t = \frac{3\pi}{2\omega}$, we have the maximum displacement into the K_1 region:

$$K_1(-x + x_0) = K_1(-a_1 + b_4 + c) = -m\omega^2 a_1 - 16m\omega^2 b_4.$$

Solving these two expressions for a_1 and b_4, we see that:

$$\begin{aligned} a_1 &= \frac{K_1(K_2 - 16m\omega^2) + K_2(K_1 - 16m\omega^2)}{(K_1 - m\omega^2)(K_2 - 16m\omega^2) + (K_2 - m\omega^2)(K_1 - 16m\omega^2)} c, \\ b_4 &= \frac{(K_2 - K_1)m\omega^2}{(K_1 - m\omega^2)(K_2 - 16m\omega^2) + (K_2 - m\omega^2)(K_1 - 16m\omega^2)} c. \end{aligned} \tag{14}$$

A survey of (14) shows that the denominators in both expressions vanish when

$$\omega^2 = \frac{17}{64}\frac{K_1 + K_2}{m} \pm \sqrt{\left(\frac{17}{64}\right)^2 \left(\frac{K_1 + K_2}{m}\right)^2 - \frac{K_1 K_2}{16m^2}}. \tag{15}$$

The physcial interpretation is therefore that the solution fails at these two values of ω. From a physical point of view, however, the failures are not of the same kind. Thus, at the larger value of ω, given by (15), we have a real pseudo-resonance condition, but the infinite amplitudes corresponding to the smaller value of ω admit of no physical interpretation.

An approximate solution of (15) gives:

$$\omega^2 = \begin{cases} \frac{34}{64}\frac{K_1 + K_2}{m} - \frac{2K_1K_2}{17m(K_1 + K_2)} \\ \frac{2K_1K_2}{17m(K_1 + K_2)}. \end{cases} \tag{15a}$$

Incidentally, the lower value of ω from (15) or (15a) is very near the value of ω at which the numerator of the expression for a_1 vanishes; this occurs when

$$\omega^2 = \frac{K_1K_2}{8m(K_1 + K_2)}. \tag{16}$$

When K_1 approaches K_2, the expressions (14) for a_1 and b_4 reduce to the trivial solution.

In concluding the discussion, it can be said that the approximate solution involving two terms admits of no physical interpretation in a small region of ω in the neighborhood given by (16), but there is no reason to discredit unduly the accuracy of the solution in the region of pseudo-resonance.

If four terms instead of two had been used, a great deal of apparent accuracy would have been lost in the region of small values of ω by the presence of three regions of ω at which physical interpretations are lacking.

Special Case II.

If the restoring force curve is given by Fig. 3, an approximate solution, involving two terms, can readily be obtained if we assume that the maximum displacements occur at $\pi/2\omega$ and at $3\pi/2\omega$. A priori it is to be expected that the displacement may be described fairly well by equation (13), so that the equations for determining a_1 and b_4 become:

$$f(x - x_0) = \begin{cases} f_1(a_1 - b_4 - c) = m\omega^2 a_1 - 16m\omega^2 b_4, \\ f_2(-a_1 + b_4 + c) = -m\omega^2 a_1 - 16m\omega^2 b_4. \end{cases} \tag{17}$$

Since $f_1(a_1 - b_4 - c)$ and $f_2(-a_1 + b_4 + c)$ are readily expressed analytically, a graphical determination of the constants is not necessary. We must, however, recognize three stages of motion; first, when $x - x_0$ is smaller than δ_1, the trivial solution then holds; second, when $x - x_0$ is greater

than δ_1, but smaller than δ_2; and third, when $x - x_0$ is greater than δ_2. If we determine a_1 and b_4 for the lastly mentioned stage, we get:

$$a_1 = \frac{(K_2 - K_1)(\delta_1 + \delta_2) \pm 2K_2c}{2(K_2 - m\omega^2)},$$
$$b_4 = \frac{(K_2 - K_1)(\delta_2 - \delta_1)}{2(K_2 - 16m\omega^2)}. \tag{18}$$

It is seen that the expression for a_1 is identical with the expression for a_1 of the single term solution for the symmetrical case, equation (7), if $\delta_1 + \delta_2 = 2\delta$. Moreover, since the expression for b_4 vanishes when $\delta_1 = \delta_2$, we are led to believe that the two term solution for the system with unsymmetrical restoring force cannot be expected to give more accurate results than the single term solution for the symmetrical case. The value of b_4 informs us about the difference of the maximum displacements on either side of the static equilibrium position. The total swing is, however, equal to a_1, unless b_4 be relatively large. When $\omega = \frac{1}{4}\sqrt{K_2/m}$, the solution has no physical interpretation.

If we determine a_1 and b_4 for the second stage, i.e., when $x - x_0$ is greater than δ_1, but smaller than δ_2, we find that:

$$a_1 = \frac{[(K_2 - K_1)\delta_1 \pm K_2c](K_1 - 16m\omega^2) \pm K_1c(K_2 - 16m\omega^2)}{(K_1 - m\omega^2)(K_2 - 16m\omega^2) + (K_2 - m\omega^2)(K_1 - 16m\omega^2)},$$
$$b_4 = \frac{-[(K_2 - K_1)\delta_1 \pm K_2c](K_1 - m\omega^2) \pm K_1c(K_2 - m\omega^2)}{(K_1 - m\omega^2)(K_2 - 16m\omega^2) + (K_2 - m\omega^2)(K_1 - 16m\omega^2)}. \tag{19}$$

Infinite values of a_1 are obtained for the two values of ω given by equations (15) or (15a). The constant b_4 also becomes infinite when $\omega = \frac{1}{4}\sqrt{K_2/m}$. The amplitude a_1 is zero when

$$\omega^2 = \frac{K_1[(K_2 - K_1)\delta_1 \pm 2K_2c]}{16m[(K_2 - K_1)\delta_1 \pm (K_2 + K_1)c]} \tag{20}$$

and it can easily be shown that this value of ω as well as the lower value of ω from equation (15) lies in the neighborhood of $\frac{1}{4}\sqrt{K_2/m}$. The higher value of ω given by (15) is, however, never reached, on account of $x - x_0$ entering into the stage described by (18).

General Case.

An approximate two term solution may be readily obtained graphically, or semi-graphically, for a general case of unsymmetrical restoring force of the type shown in Fig. 4*b* if it be assumed that the maximum displacements occur when the time is $\pi/2\omega$ and $3\pi/2\omega$. Under these conditions, the displacement will be represented by

$$x = a_1 \sin \omega t - b_n \cos n\omega t, \tag{13a}$$

in which n is an even number. Equations (9), (11), and (12) then give:

$$\begin{aligned} f_1(a_1 \pm b_4 \mp c) &= m\omega^2(a_1 \pm n^2 b_n) \\ &= m\omega^2(a_1 \pm n^2 b_n \mp c) \pm m\omega^2 c, \\ f_2(-a_1 \pm b_4 \pm c) &= m\omega^2(-a_1 \pm n^2 b_n) \\ &= m\omega^2(-a_1 \pm n^2 b_n \pm c) \mp m\omega^2 c. \end{aligned} \tag{21}$$

The upper sign of b_n is to be taken if $n = 2$, 6, 10, etc., and the upper sign of c if we assume "in phase" motion. Moreover, for graphical purposes it is convenient to represent f_1 and f_2 in the positive quadrant, so that if we let $n = 2$ and consider "in phase" motion, equations (21) become:

$$\begin{aligned} f_1(a_1 + b_2 - c) &= m\omega^2(a_1 + 4b_2 - c) + m\omega^2 c, \\ f_2(a_1 - b_2 - c) &= m\omega^2(a_1 - 4b_2 - c) + m\omega^2 c. \end{aligned} \tag{21a}$$

The graphical solution for finding a_1 and b_2 now consists of the following steps:

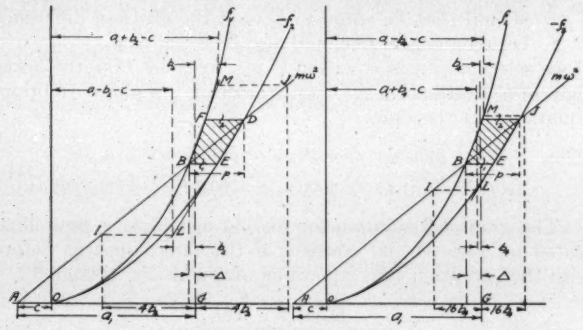
1st) Draw functions f_1 and $-f_2$ to a convenient scale, Fig. 6*a*.

2d) Lay off the distance c from O to A.

3d) Through A draw a straight line of slope $m\omega^2$; this line cuts the f_1 and $-f_2$ curves at B and D.

4th) Project B on the $-f_2$ curve, and project D on the f_1 curve, thereby forming the cross hatched, four cornered area $BEDF$.

FIG. 6a. FIG. 6b.



Graphical solution involving general unsymmetrical restoring force curve of Fig. 4b, also equations (21a), (22), and (23).

Graphical solution involving general unsymmetrical restoring force curve of Fig. 4b, also equations (21b), (22a), and (23a).

5th) On the assumption that the curve segments BF and ED may be considered straight lines, let the projection of BD on the abscissa be denoted by p; also let $BE = l_1$; and let $FD = l_2$, then compute the distance Δ from the relation:

$$\Delta = \frac{pl_1(3p - 4l_2)}{3p(l_1 + l_2) - 8l_1l_2}. \tag{22}$$

6th) Add Δ to the projection of B on the abscissa, thereby obtaining G. The distance OG is equal to $a_1 - c$; the distance AG is equal to the coefficient a_1.

7th) The value of b_2 may be found from the relation

$$b_2 = \frac{\Delta l_2}{3p - 4l_2}. \tag{23}$$

If it be desired to make a graphical check of the calculations, proceed as follows:

a) Lay off $\pm 4b_2$ from G and project on the straight line ABD, thereby obtaining the points I and J.

b) Project I on the straight line passing through ED at L, and project J on the straight line passing through BF

at M. The difference in the abscissas of M and G should then be equal to b_2, and the abscissa difference between G and L should also be equal to b_2.

If we select $n = 4$ in equation (13a), we must take the lower signs of b_n in equation (21). Considering "in phase" motion, equations (21) become:

$$\begin{aligned} f_1(a_1 - b_4 - c) &= m\omega^2(a_1 - 16b_4 - c) + m\omega^2 c, \\ f_2(a_1 + b_4 - c) &= m\omega^2(a_1 + 16b_4 - c) + m\omega^2 c. \end{aligned} \tag{21b}$$

The graphical solution for finding a_1 and b_4 is now illustrated on Fig. 6*b* and consists of the same steps as before with the exception that equations (22) and (23) become:

$$\Delta = \frac{pl_1(15p - 16l_2)}{15p(l_1 + l_2) - 32l_1l_2}, \tag{22a}$$

$$b_4 = -\frac{\Delta l_2}{15p - 16l_2}. \tag{23a}$$

A graphical check of the calculations is also shown in the figure.

A comparison of the accuracy obtained by using b_2 and b_4 is not readily made. It is true that b_4 is much smaller than b_2, but if we had chosen $n = 8$, the value of b_8 would have been found still smaller than b_4, etc., not necessarily indicating a better approximation. A priori we should expect that the solution involving b_4 would give a closer approximation than the one using b_2, on account of the displacement curve involving b_4 fitting the physical conditions better than the one with b_2, but it must be admitted that the only general conclusion that can be made is that whatever be the selected even value of n, Δ cannot be such that the point G projects outside the cross hatched area. When f_1 is the same as $-f_2$, the solution reduces to the single term given by Professor Den Hartog for the symmetrical case.

Alternative Solutions of the Unsymmetrical Case.

Before we proceed to discuss multiple term solutions, let us consider the following expression for the displacement x

$$x = a_1 \sin \omega t - S, \tag{24}$$

in which S is a "temporary constant"; in other words, it is a function of the physical constants of the system and of ω. Of course, if we want to consider S the summation of a Fourier Series, we are at liberty to do so.

Remembering that we are interested in approximate solutions only, we proceed as follows:

Let

$$f(x - x_0) = A_1 \sin \omega t - \varphi(t). \tag{25}$$

Substituting (24) and (25) into (1), we obtain

$$-m\omega^2 a_1 \sin \omega t + A_1 \sin \omega t - \varphi(t) = 0. \tag{26}$$

We see that (26) is satisfied by the usual relation

$$A_1 = m\omega^2 a_1$$

if the function $\varphi(t)$ is zero. Let us assume that $\varphi(t)$ is zero, and then proceed to determine the constants a_1 and S at the times $\pi/2\omega$ and $3\pi/2\omega$ when the displacements are maxima. This gives:

$$\begin{aligned} f_1(a_1 - S - c) &= m\omega^2 a_1, \\ f_2(-a_1 - S + c) &= -m\omega^2 a_1, \end{aligned} \tag{27}$$

Instead of equations (14), the Special Case I now gives:

$$\begin{aligned} a_1 &= \frac{2K_1K_2c}{K_2(K_1 - m\omega^2) + K_1(K_2 - m\omega^2)}, \\ S &= \frac{K_1(K_2 - m\omega^2) - K_2(K_1 - m\omega^2)}{K_2(K_1 - m\omega^2) + K_1(K_2 - m\omega^2)}\, c, \end{aligned} \tag{28}$$

and we note with satisfaction that (14) and (28) are identical for large values of ω and that infinite amplitudes are obtained only for one value of ω, namely the one for which a physical interpretation exists:

$$\omega^2 = \frac{2K_1K_2}{m(K_1 + K_2)}. \tag{29}$$

It is easily shown that the larger value of ω from (15) is a neighbor to the value of ω from (29) if K_1 and K_2 are not very different.

Applying equations (27) to the Special Case II, we now obtain:

$$a_1 = \frac{(K_2 - K_1)(\delta_1 + \delta_2) \pm 2K_2 c}{2(K_2 - m\omega^2)},$$
$$S = \frac{(K_2 - K_1)(\delta_2 - \delta_1)}{2K_2}. \tag{30}$$

A comparison of this result with (18) shows that the coefficients a_1 are exactly alike, but b_4 and S are somewhat different.

When the intermediate stage of Case II is considered, we obtain from (27):

$$a_1 = \frac{(K_2 - K_1)K_1\delta_1 \pm K_1K_2c}{(K_1 - m\omega^2)K_2 + (K_2 - m\omega^2)K_1},$$
$$S = \frac{(K_2 - K_1)\delta_1(K_1 - m\omega^2) \pm K_1c(K_2 - m\omega^2)}{(K_1 - m\omega^2)K_2 + (K_2 - m\omega^2)K_1}. \tag{31}$$

When compared with the result formerly obtained, equations (19), the agreement in this stage is seen to be less satisfactory than before.

A graphical solution of equations (27) is especially simple. Referring to Fig. 7, the steps are as follows:

1st) Draw f_1 and $-f_2$ to a convenient scale.
2d) Draw the dotted curve f_m in such a way that it bisects the abscissa difference of f_1 and $-f_2$.
3d) Lay off the distance c from O to A.
4th) Through A draw a straight line with slope $m\omega^2$; this line cuts f_m at M; AG is then equal to the coefficient a_1.
5th) Through M draw a line parallel to the abscissa cutting f_1 at B and $-f_2$ at E. BM is equal to the constant S.

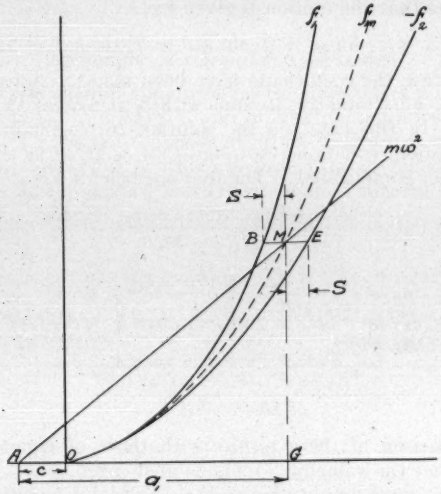
Multiple Term Solutions.

Solutions have been obtained for the special cases already considered by assuming that the displacement x is given by:

$$x = a_1 \sin \omega t + a_3 \sin 3\omega t - b_2 \cos 2\omega t - b_4 \cos 4\omega t \tag{32}$$

and the coefficients have been adjusted to agree with f_1 at $\pi/4\omega$, $\pi/2\omega$, and $3\pi/4\omega$, and with f_2 at $5\pi/4\omega$, $3\pi/2\omega$, and $7\pi/4\omega$. Only the solution corresponding to the large motion given by equations (18) is sufficiently simple to be of any value

FIG. 7.



Graphical solution involving general unsymmetrical restoring force curve of Fig. 4*b*, also equation (27).

in making a comparison of the two term and four term, solutions. The four coefficients are:

$$
\begin{aligned}
a_1 &= \frac{(K_2 - K_1)1.206(\delta_1 + \delta_2) \pm 2K_2c}{2(K_2 - m\omega^2)}, \\
a_3 &= \frac{(K_2 - K_1).206(\delta_1 + \delta_2)}{2(K_2 - 9m\omega^2)}, \\
b_2 &= \frac{(K_2 - K_1)(\delta_1 - \delta_2)}{K_2 - 4m\omega^2}, \\
b_4 &= \frac{(K_2 - K_1)(\delta_1 - \delta_2)}{2(K_2 - 16m\omega^2)}.
\end{aligned}
\tag{33}
$$

It is seen that b_4 is the only coefficient that remains unchanged in equations (18) and (33).

Solutions have also been obtained for the special cases by assuming that the motion is given by:

$$x = a_1 \sin \omega t + a_3 \sin 3\omega t + a^5 \sin 5\omega t - S. \qquad (34)$$

In this case the coefficients have been made to agree with f_1 at $\pi/3\omega$, $\pi/2\omega$, and $2\pi/3\omega$, and with f_2 at $4\pi/3\omega$, $3\pi/2\omega$, and $5\pi/3\omega$. In this case too the solution corresponding to the large motion, equations (30), and (33), is given as the others are rather complicated. The four coefficients are:

$$\begin{aligned} a_1 &= \frac{(K_2 - K_1)1.156(\delta_1 + \delta_2) \pm 2K_2 c}{2(K_2 - m\omega^2)}, \\ a_3 &= \frac{(K_2 - K_1).156(\delta_1 + \delta_2)}{2(K_2 - 9m\omega^2)}, \\ a_5 &= 0, \\ S &= \frac{(K_2 - K_1)(\delta_1 + \delta_2)}{2K_2}. \end{aligned} \qquad (35)$$

A comparison of these results with those of equation (30) shows that the value of S remains unchanged.

If the total amplitude, $a_1 + a_3$, given by equations (33) and (35) be compared with the total amplitude, a_1, of the two term solutions, equations (18) and (30), we see that they are all the same when

$$\omega = \frac{\sqrt{2}}{3}\sqrt{\frac{K_2}{m}} = .472\sqrt{\frac{K_2}{m}}.$$

In concluding the theoretical part of the investigation we are obliged to admit that an evaluation of the accuracy obtained by the two term and four term solutions of the two special cases is not possible without the existence of an exact solution for these cases. In Special Case I we have the choice of equations (14) or (28). In Special Case II the large motion stage gives us the choice of equations (18) and (30), while the intermediate motion stage is described by equations (19) or (31). Moreover, if a four term solution is desired, we have equations (33) or (34) for the large motion stage.

The question naturally arises, how well does the single term solution of the symmetrical case and the two term

solution of the unsymmetrical case fit the actual phenomena? The experiments following this discussion seem to give a satisfactory answer to the question.

DESCRIPTION OF EXPERIMENTAL APPARATUS.

The experimental apparatus for the investigation is shown diagrammatically on Fig. 8 and on the photographs Figs. 9 and 10. The model represented in the diagram is mounted on the platform, Fig. 9, which furnishes the so-called ground motion. The actuating mechanism of the platform, Fig. 10, involves a heavy flywheel with an adjustable crank and a

FIG. 8.

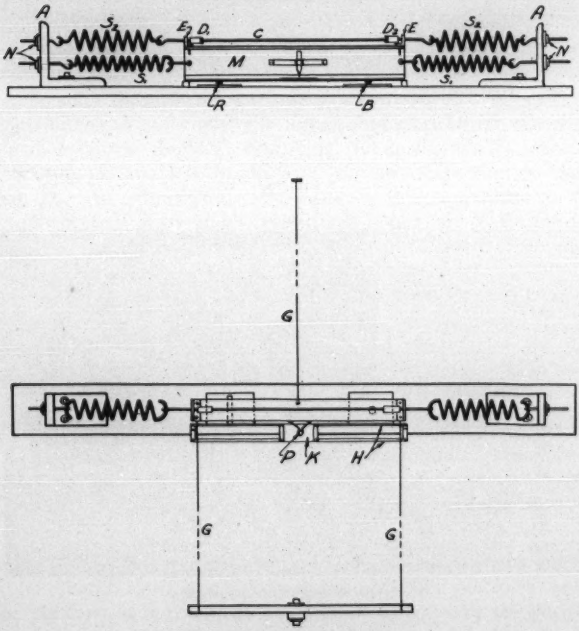
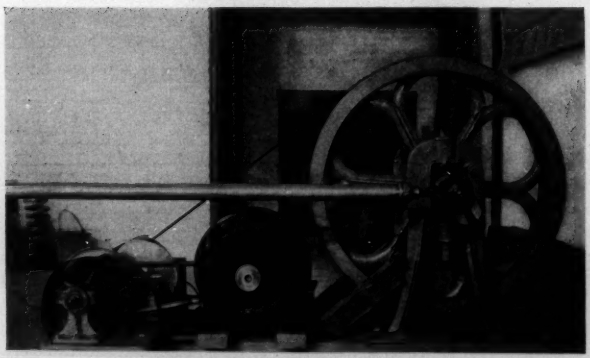


Diagram of experimental arrangement.

FIG. 9.

Photograph showing model mounted on small shaking table.

FIG. 10.



Photograph showing actuating mechanism.

very long connecting rod. An induction motor with a mechanical speed regulating device serves as the prime mover.

Referring to the diagram of the model, Fig. 8, it will be seen that the mass M with its two systems of springs constitutes a self contained system with the anchoring supports A mounted on a heavy plank. The mass consists of a section of a 4 inch I beam, 25 inches long, and symmetrically loaded. The lower side of the I beam is planed so as to roll smoothly on its two supports, the roller R and the ball B. In order to avoid transverse motion of the mass, the strings G are attached, these strings being long enough (8 feet) so that the longitudinal restoring force due to a change in their length caused by a longitudinal displacement of M can be neglected.

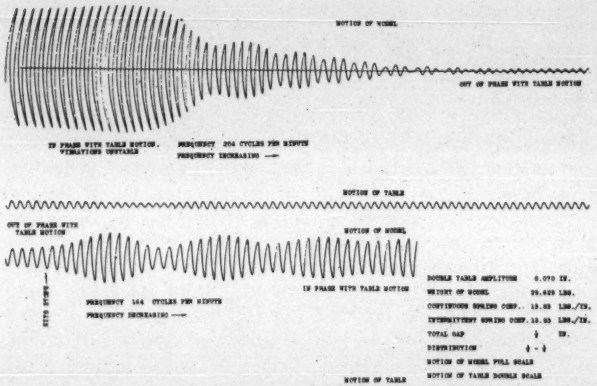
The longitudinal motion of the mass is restricted by the two sets of springs. The first of these, S_1, is always connected with the mass, and therefore represents the spring factor K_1. The springs S_2 are always connected to the rod C which is free to slide through holes in the small angles E. When the mass moves to the right, the adjustable stop D_1 on the rod is struck by the angle E, rendering the spring set S_2 active. A motion to the left produces a similar effect due to the stop D_2. In other words, the mass is free to vibrate as a simple system with small amplitude about its static equilibrium position, but as soon as the displacements become greater than the gaps, the second spring set enters into action, and together with the first spring set now represents the spring factor K_2.

RECORDING DEVICES.

Since the ground motion amplitude depends mainly on the crank throw and is therefore kept reasonably constant during an experiment, a record of the relative maximum amplitude of the mass with respect to the ground or platform is sufficient for checking the theory, inasmuch as no account is taken of wave form. In order to obtain a mechanical record with a minimum disturbance of the steady motion, a small recording platform K, Fig. 8, is mounted on two slender steel springs H fastened to the plank next to the mass M. When a record is to be taken, a card is attached to the recording platform, and the platform is temporarily elevated so as to make the pencil P, which is fixed to the mass, touch the card. All of the tests have been recorded in this way.

In addition to the above method of recording, it was necessary, or at least desirable, to have a time record of the motion when instability occurred. For this purpose, the mass was connected by means of a piano wire to a pen tracing a record on a chronograph drum, and the ground, or platform, was connected similarly to a second pen. The connections are such that the absolute motion of the mass is recorded, while, for the sake of clearness, the absolute motion of the ground was magnified two times. Figure 11 shows typical time displace-

FIG. 11.


MOTION OF WHEEL
OUT OF PHASE WITH TABLE MOTION
IN PHASE WITH TABLE MOTION.
VIBRATIONS UNSTABLE
FREQUENCY 204 CYCLES PER MINUTE
FREQUENCY INCREASING —
MOTION OF TABLE
OUT OF PHASE WITH TABLE MOTION
MOTION OF WHEEL
IN PHASE WITH TABLE MOTION
FREQUENCY 144 CYCLES PER MINUTE
FREQUENCY INCREASING —
DOUBLE TABLE AMPLITUDE 0.070 IN.
WEIGHT OF WHEEL 29.625 LBS.
CONTINUOUS SPRING CONST.. 15.85 LBS./IN.
INTERMITTENT SPRING CONST. 15.85 LBS./IN.
TOTAL GAP ¼ IN.
DISTRIBUTION ¼ - ¼
MOTION OF WHEEL FULL SCALE
MOTION OF TABLE DOUBLE SCALE
MOTION OF TABLE

Record showing transition from in phase to out of phase motion, also from out of phase to in phase motion. The recorded curve is far from sinusoidal due to angularity of recording pen.

ment records of the system for increasing as well as for decreasing frequencies of the motion.

All of the experimental work was carried out by Mr. Jespersen, and has been recorded in detail in his thesis for the Engineer's Degree. Altogether 72 tests have been made with 32 different combinations of K, δ, and c. The elements of the combinations are as follows:

$$\left.\begin{matrix} K_2 = 2K_1 \\ K_2 = 3K_1 \end{matrix}\right\} \left\{\begin{matrix} \delta_2 = \delta_1 \\ \delta_2 = 3\delta_1 \end{matrix}\right\} \left\{\begin{matrix} \delta_1 + \delta_2 = 1'' \\ \delta_1 + \delta_2 = \tfrac{1}{2}'' \end{matrix}\right\} c = \left\{\begin{matrix} .020'' \\ .035'' \\ .050'' \\ .060'' \end{matrix}\right.$$

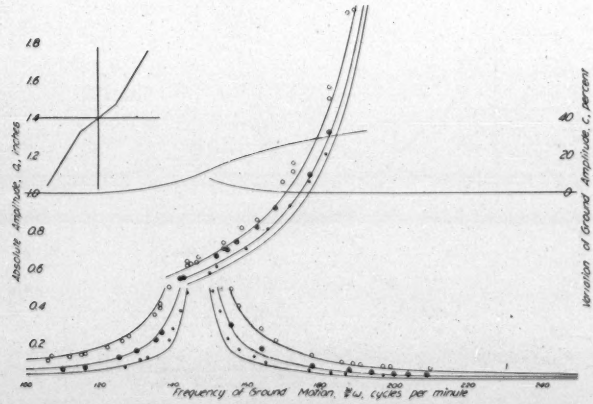
Each test involved from 24 to 32 frequency changes, and at each frequency at least two records of maximum amplitude were taken.

DISCUSSION OF RESULTS AND CONCLUSIONS.

The observed, absolute amplitudes have been plotted on the theoretical curves for the single term solution, equation (7), of the symmetrical case, and for the two term solution, equations (18) and (19), of the unsymmetrical case; Figs. 12 to 19 inclusive give the results.

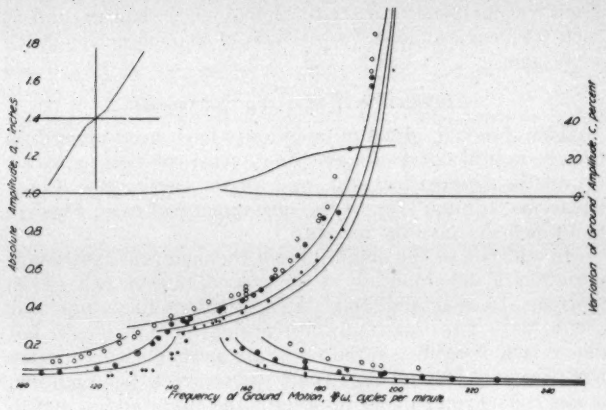
In addition to the amplitudes of the mass, the percentage variation of the amplitude of the ground motion, c, has been plotted. It was originally intended that this amplitude should have been kept constant during an experiment, but due to lack of rigidity in the actuating mechanism a variation was present. Instead of attempting to correct this condition, it was considered simpler to observe the amplitude, c, and to use these values in the equations from which the theoretical curves have been computed.

FIGS. 12 TO 19 INCLUSIVE.



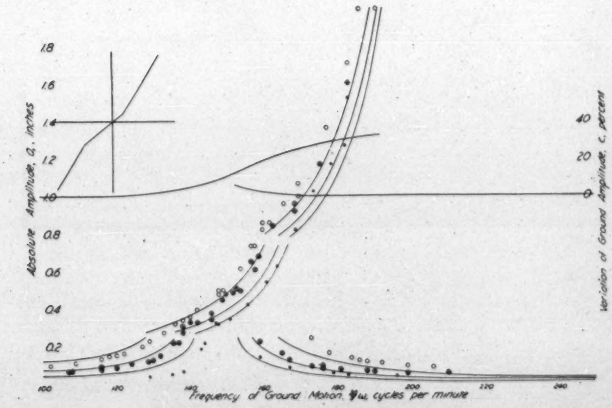
m = 0.0664 lbs./386; K_2 = 33.8 lbs./inch; K_1 = 15.8 lbs./inch; δ_1 = 0.50 inch; δ_2 = 0.50 inch.

○ C = 0.060; ◎ C = 0.035; ● C = 0.020.



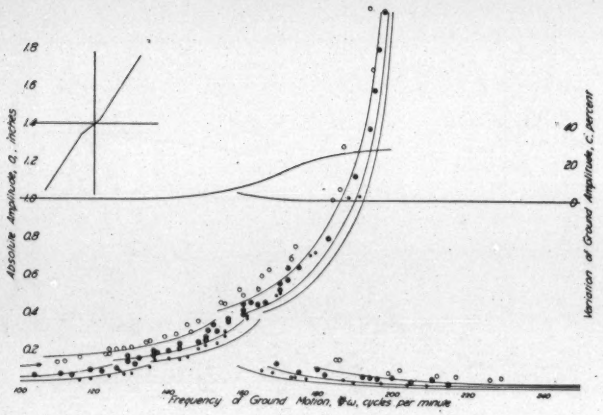
m = 0.0664 lbs./386; K_2 = 33.8 lbs./inch; K_1 = 15.8 lbs./inch; δ_1 = 0.25 inch; δ_2 = 0.25 inch.

○ C = 0.060; ◎ C = 0.035; ● C = 0.020.



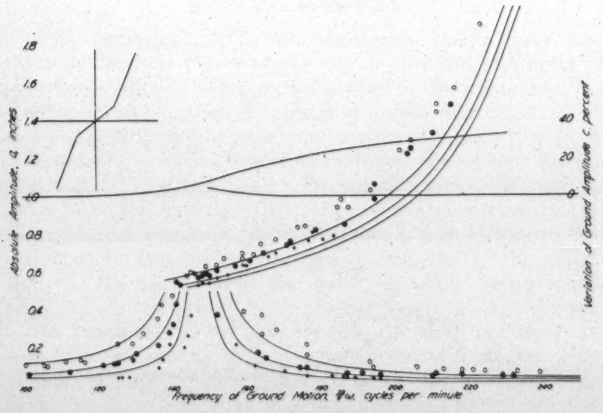
m = 0.0664 lbs./386; K_2 = 33.8 lbs./inch; K_1 = 15.8 lbs./inch; δ_1 = 0.25 inch; δ_2 = 0.75 inch.

○ C = 0.060; ◎ C = 0.035; ● C = 0.020.


Absolute Amplitude, A, inches
Variation of Ground Amplitude, C, percent
Frequency of Ground Motion, $\frac{\phi}{2\pi}$, cycles per minute

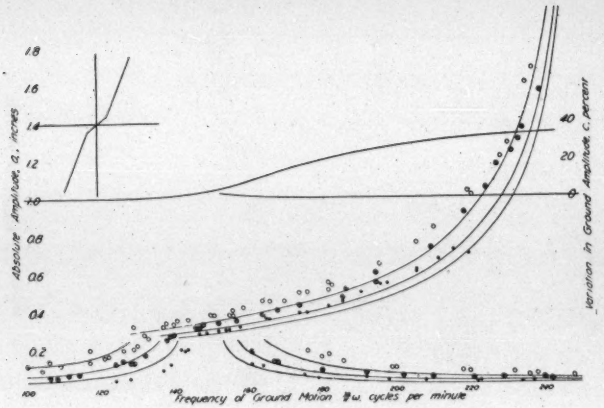
$m = 0.0664$ lbs./386; $K_2 = 33.8$ lbs./inch; $K_1 = 15.8$ lbs./inch; $\delta_1 = 0.125$ inch; $\delta_2 = 0.375$ inch.

○ C = 0.060; ◎ C = 0.035; ● C = 0.020.


Absolute Amplitude, A, inches
Variation of Ground Amplitude, C, percent
Frequency of Ground Motion, $\frac{\phi}{2\pi}$, cycles per minute

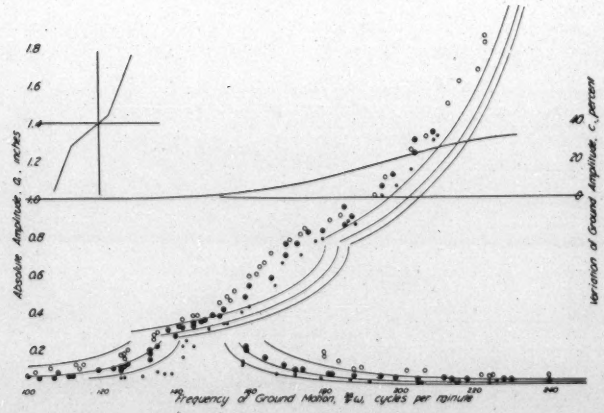
$m = 0.0664$ lbs./386; $K_2 = 49.0$ lbs./inch; $K_1 = 15.8$ inches; $\delta_1 = 0.50$ inch; $\delta_2 = 0.50$ inch.

○ C = 0.060; ◎ C = 0.035; ● C = 0.020.



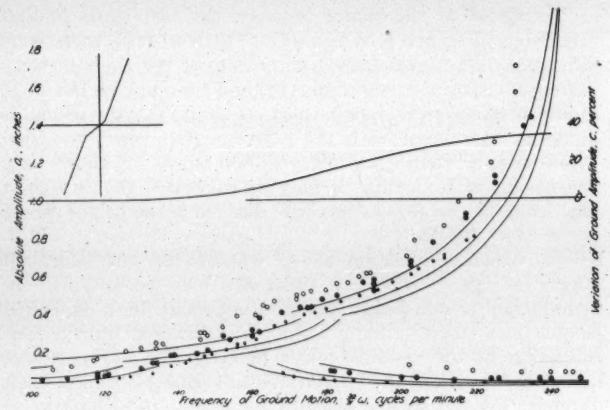
m = 0.0664 lbs./386; K_2 = 49.0 lbs./inch; K_1 = 15.8 lbs./inch; δ_1 = 0.25 inch; δ_2 = 0.25 inch.

○ C = 0.060; ◎ C = 0.035; ● C = 0.020.



m = 0.0664 lbs./386; K_2 = 49.0 lbs./inch; K_1 = 15.8 lbs./inch; δ_1 = 0.25 inch; δ_2 = 0.75 inch.

○ C = 0.060; ◎ C = 0.035; ● C = 0.020.



m = 0.0664 lbs./386; K_2 = 49.0 lbs./inch; K_1 = 15.8 lbs./inch; δ_1 = 0.125 inch; δ_2 = 0.375 inch.

○ C = 0.060; ◎ C = 0.035; ● C = 0.020.

Experimental verification of theory.

The theoretical curves are consistently a little lower than the experimental points, especially in the neighborhood of pseudo-resonance. This may be due to the fact that the guiding strings G, Fig. 8, exercised a restoring effect which was not negligible when the amplitude was large.

Contrary to usual expectations it was found that for the symmetrical case the single term solution given by equation (7) is in better agreement with the observations than the two term solution, equation (7a). This fact is therefore a corroboration of Professor Den Hartog's remark that: "the golden rule of 'the more work, the more accuracy' is no longer true."

In a similar way the agreement in the unsymmetrical case was fully as satisfactory when the two term solutions, equations (18) and (30), were used as when the more elaborate four term solutions, equations (33) and (34), were employed. It may therefore be concluded that in the cases observed the first approximations constitute satisfactory solutions, at least from a practical point of view.

In regard to the choice between the two types of first approximations, given by equations (13) and (24), there is no difference as far as the total amplitude of the stage of large motion is concerned, equations (18) and (30), but for the stage of intermediate motion, equations (19) and (31), the second is in better agreement with the experiments. Since equation (31) is simpler than equation (19), and since (31) shows only one resonance frequency, it may be suggested that equation (24) from which (30) and (31) are derived is the better choice in the cases considered.

The original question as to whether or not beneficial effects in regard to seismic resistance may be obtained by introducing a non-linear restoring element in a so-called flexible first story building cannot be answered from a consideration of the results of this investigation. Theoretical first approximations with experimental verifications have been obtained for the steady state of motion of the single mass system, but the superstructure of a building does not constitute a single mass, and the frequency of the ground motion is not constant during an earthquake for a sufficiently long time to bring the building into the steady state of motion. It is suggested, however, that the initial step has been made toward a solution of the seismic problem, and that subsequent steps, on account of the inherent complexity of the problem, practically resolve themselves into experiments with models satisfying the conditions for dynamic similitude. This work is now in progress at the Vibration Laboratory at Stanford University.

STANFORD UNIVERSITY, CALIF.,
December, 1933.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

DESIGN OF TRANSFORMERS FOR AMATEURS.

Among the many requests which the Bureau receives for technical information are a considerable number from amateurs who require design data on power transformers for various purposes. It is not practicable to spend the time of the staff in working out individual design problems which are not of public interest. For a number of years, such requests have been partially covered by sending a very short typewritten memorandum on general principles of transformer design. The use of this information, however, involved calculations with which many amateurs are not familiar, and left rather too much to the judgment of the individual.

In seeking for means to improve this memorandum, the possibilities of using curves or graphs to reduce computation were considered. By the use of logarithmic and semilogarithmic cross-section paper it has been possible to reduce all of the graphs except one to straight lines, and to reduce the computation to a very small and simple matter. For example, the designer can tell at a glance how much iron and how much copper will be needed for a transformer of any rated output between 1 voltampere and 10 kilovoltamperes (that is, 1 watt and 10 kilowatts, at unity power factor); how large must be the cross section of the core; how many turns of wire must be used per volt; the sizes of wire; the regulation; the values of the iron loss and the full-load copper loss; and the efficiency at full load. These labor-saving graphs are the basis of Circular C408 which has just been issued by the Bureau under the title, "Information for the Amateur Designer of Transformers for 25- to 60-Cycle Circuits."

The circular includes definitions of terms used in connection with transformers, and states very briefly the mathematical relations on which the design procedure is based. It gives information on kinds of magnet wire suitable for use

* Communicated by the Director.

in making transformers, and outlines methods of constructing the windings and of forming the core. Special information is given for cases which present unusual difficulties, namely, windings for large currents at a low voltage, windings with low-voltage taps, and transformers to have good regulation with inductive loads at low power factor. Although the method of design is intended primarily for the usual frequency of 60 cycles, it may be readily modified so as to be applicable to 25 cycles and to 50 cycles.

As a result of correspondence between the Bureau and the makers of magnet wire and of transformer steel sheets, it has been possible to include in the circular a list of names and addresses of concerns willing to sell small quantities of material to the amateur constructor. Suggestions are given in the circular as to local sources of wire and of insulating varnish.

Copies of this circular are available from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 5 cents each.

ULTRAVIOLET SOLAR RADIATION INTENSITIES.

Two errors occur in this item on pages 79 and 80 of Technical News Bulletin No. 220 (August 1935). In the fourth paragraph on page 79 "8 km. (7,000 ft.)" should read "2 km. (7,000 ft.)." In the second paragraph on page 80 "(at 2900 to 300 A)" should read "(at 2900 to 3000 A)."

RELATIVE DURABILITY OF OPTICAL GLASS.

Numerous methods have been recommended for determining the relative durability of optical glasses and in most cases the results are qualitative and depend on individual judgment or area of the specimen. In connection with the Bureau's work on optical glass it seemed that the attack by acid and alkaline solutions might indicate durability and that such attack might be measured by interferometry.

Small optical flats of medium flint, borosilicate crown and light barium crown (indices of refraction 1.620, 1.517 and 1.574, respectively) were partially immersed in 5 per cent. solutions of either caustic soda or hydrochloric acid, the

surface of the solution covered with a high grade mineral oil and the assembly heated in a constant temperature water bath for varying periods of time. The specimens were removed from the solution, washed, dried and examined under an optical flat with a modified Pulfrich viewing apparatus. The line of demarcation corresponding to the lower surface of the oil covering the solution was very sharp and the accompanying shift in the fringes was very uniform. The amount of the shift could be readily measured.

In 5 per cent. caustic soda at 80° C. for 3 hours the shift for the light barium crown, borosilicate crown and medium flint was 3.25, 3 and 2.5 fringes, respectively. Similarly, in 5 per cent. hydrochloric acid the shifts were 0.8, 0.2 and "just detectable."

It was also found that the attack at constant temperature was directly proportional to the time and between 30° and 80° C., the time being constant, the attack was doubled for every 10° increase in temperature.

For a first relative comparison a sample of commercial window glass was treated in the same way and the shift was 1.25 fringe in the alkaline solution, but in hydrochloric acid there was no detectable attack in 24 hours.

The results are quite reproducible and since this method of measuring attack is definite and independent of the surface exposed, the method seems to warrant further consideration.

DETERMINATION OF REFRACTIVE INDEX BY IMMERSION.

The mineral constituents of rocks are commonly identified by their optical properties, chief of which are three refractive indices. These indices correspond with vibration of light parallel to three optical directions of a crystal. It has long been hoped that these same methods, which are rapid, convenient, and non-destructive of samples, might be used in the identification of chemical compounds. The number of compounds between which it may be necessary to choose is immensely larger, however, than the total number of minerals. It is fundamentally important, therefore, that the ultimate attainable accuracy of the methods be known if such application is to be made extensively. In a recent investigation at

the Bureau, reported in the September *Journal of Research*, RP829, it was found that under different conditions, presumed to be equally satisfactory, the errors resulting from uncertainty in matching the index of a liquid to that of a crystal may easily differ by a factor of ten.

Another type of error may originate from the birefringence of the crystals. When a crystal is strongly birefringent, errors from this cause may become very large indeed. Under one set of conditions the error in determining the extraordinary refractive index of mercuric cyanide was 0.0266, but under other conditions it was not detectable.

The ordinary method of oblique illumination should never be employed unless inexact results will suffice. Other methods should be used with a full realization of the errors to which they are subject.

INFRARED ABSORPTION SPECTRA OF PLANT AND ANIMAL TISSUE.

Supplementing a previous report on plant pigments (*J. Research N. B. S.*, 11, 703 (November 1933) (RP617)), a paper in the September number of the *Journal of Research* (RP830) describes new data on the infrared absorption spectra of plant and animal substances (cellulose and proteins) as found in nature and, in some cases, in the form of pure material prepared in the laboratory.

The data were obtained by means of a mirror spectrometer, portable vacuum thermopile, and ironclad Thomson galvanometer, described in previous publications. The region of the infrared spectrum extending to 15 microns was examined.

Outstanding among the plant substances examined are samples of pure rubber, and associated materials—styrene, indene, polystyrene, and polyindene.

Among the cellulose materials examined are: cellophane (commercial preparation), onion skin, translucent membrane of pith of pokeweed, the seed septum of moonwort, and the seed wing of the cotton tree.

Among the protein materials examined are: dried specimens of chitin (the outer integument of insects), pith of

feathers, air bladder of fish, bat's wings, film of dried egg albumin, commercial gelatin, lining of egg shell, and wing of a dragon fly.

Among the miscellaneous substances reported upon are polyvinyl acetate and polyvinyl chloracetate resins, glyptal resin, shellac, and a group of substances of interest in connection with the Raman effect.

HEATS OF VAPORIZATION OF EIGHT GASOLINES.

One of the hazards of aviation results from the fact that when an airplane flies through a fog or through air of high humidity at temperatures near the freezing point of water, there is a tendency for ice to form around the engine throttle or in other parts of the intake system as a result of cooling produced by evaporation of the fuel. This ice has a throttling effect on the engine, resulting in a loss of power. In extreme cases the throttling may be sufficient to stall the engine.

In RP825 in the September number of the *Journal of Research* it is shown that the rate of formation of ice under given conditions will be proportional to the product of heat of vaporization and the rate of vaporization of the fuel in the carburetor. The investigation reported in this paper was undertaken for the purpose of obtaining data on the heats of vaporization of a number of fuels. The values obtained for the heats of vaporization of the various fuels are so nearly the same that it is concluded that large differences in the rate of formation of ice with different fuels cannot be attributed to differences in heat of vaporization, but must be due to a large extent to differences in the completeness of vaporization in the carburetor, resulting from the different volatilities of the fuels.

ISOLATION AND IDENTIFICATION OF THE VOLATILE HYDROCARBONS IN A MID-CONTINENT PETROLEUM.

Precise information on the hydrocarbon constitution of a fraction of petroleum boiling between 55° and 145° C. has resulted from the work of Project No. 6 of the American Petroleum Institute conducted under the Bureau's auspices. This has been achieved by supplementing careful fractional distillation by other physical and chemical methods of

separation to obtain hydrocarbons of unquestionable purity. The identity of the compounds was established by comparison of their physical constants with those of synthetic hydrocarbons when these were obtainable. In addition to the compounds which have been actually isolated, it has been possible to predict the presence of others from the distribution of volumes and refractive indices of the constant-boiling fractions of the distillate after the removal of the predominant compounds.

The amounts of these hydrocarbons which were in the original crude oil have been estimated as carefully as possible. These amounts are expressed as percentages of the volume of the fraction boiling between 55° and 145° C. at an early stage of the distillation. Unavoidable losses which have occurred in the many operations of the analysis make the estimates of the percentages somewhat uncertain. The numbers, however, may be regarded as proportional parts and compared among themselves with considerable certainty. Thus, assuming that the hydrocarbons which have been suspected, together with those actually isolated, comprise practically all of the constituents of the fraction, 55° to 145° C., of the mid-continent oil, the amounts of the paraffin, naphthene, and aromatic hydrocarbons probably stand in the ratio 6 : 3 : 1.

Some work has been done on the fraction boiling between 145° and 180° C. but the analysis is much less complete than that between 55° and 145° C. There appears to be a larger proportion of aromatic material present but no conclusions can be drawn as to the relative content of the other classes. This project is fully reported as RP824 in the September number of the *Journal of Research*.

EFFECTS OF FUMIGANTS ON PAPER.

In RP828 in the September number of the *Journal of Research*, it is shown that the valuable records of the Government can be fumigated to rid them of book worms without danger of injury to the documents. In a recent study in the Bureau's paper section, several commercially available fumigants, known to be deadly to destructive insects often asso-

ciated with old books and documents, were found to be harmless to paper. Hence, the Government can safely use these chemicals to rid valuable records of bookworms before preserving them in the National Archives Building. In investigating the action of the gases used for killing insects, representative papers were fumigated in a gas chamber at the Bureau of Entomology and Plant Quarantine, Department of Agriculture, after which the effects of each gas were determined by chemical and physical tests of the papers. Hydrocyanic acid gas, carbon disulphide, ethylene chloride-carbon tetrachloride, ethylene oxide-carbon dioxide, and methyl formate-carbon dioxide were used without injury to the papers.

PURIFICATION OF GALLIUM.

In many chemical processes, fractional crystallization is employed as a means of purifying the material under investigation. In almost all cases, the crystals become purer every time the substance is recrystallized. This rule does not hold in the case of fractional crystallization of metallic gallium. It has been found that if gallium, containing (as impurities) small amounts of antimony, silver, gold, bismuth, columbium, cobalt, chromium, copper, mercury, iron, indium, manganese, molybdenum, nickel, osmium, lead, palladium, platinum, rhodium, ruthenium, tin, thallium, vanadium, and zinc, is subjected to fractional crystallization of the metal, all the impurities named tend to concentrate in the crystalline portion, with the following exceptions: Silver, mercury, indium, lead, and tin are concentrated in the molten residue; copper and thallium remain about equally divided between crystals and residue; zinc is dissolved by the hydrochloric acid under which the crystallization takes place, and is entirely eliminated.

It is also shown that separation from iron or platinum in excess of 0.001 per cent., from indium or lead in excess of 0.01 per cent., or from tin in excess of 0.02 per cent. by fractional crystallization of the metal is impracticable.

RP823 in the *Journal of Research* for September contains the complete account of this work.

AN EXTENSOMETER COMPARATOR.

An extensometer comparator is described in RP822 in the September number of the *Journal of Research*. This apparatus is in use in the engineering mechanics section of the Bureau for the calibration of extensometers, compressometers, dial micrometers, strain gages, and length-measuring instruments used in materials testing. In making length measurements standard steel gage blocks are used which are believed to insure an accuracy in the final measurement of about 0.00001 inch. The comparator may be used for instruments having a gage length up to 24 inches and will work equally well in a horizontal or vertical position.

TESTS OF BARE TOWER COLUMNS FOR THE GEORGE WASHINGTON BRIDGE.

By applying the principles of mechanics and of the strength of materials, engineers have developed methods of designing structures and machines which function satisfactorily. If, however, they use the available data to design a structural member several times the size of members previously built, the results have in some cases been unsatisfactory, and some structures have failed under the loads for which they were designed. This is particularly true of fabricated steel columns. The failure of the compression members (columns) of the first Quebec Bridge across the St. Lawrence River emphasized the need for tests on these important structural elements. The United States Government therefore purchased the hydraulic testing machine having a capacity of 10,000,000 lb., now in the Bureau's laboratory, which is said to be the largest testing machine in the world.

When the bridge department of the Port of New York Authority was building the George Washington (suspension) Bridge across the Hudson River it requested the Bureau to coöperate in an investigation of the strength of the large fabricated steel columns used in the towers of this bridge.

The results of this investigation are given in RP831 in the September number of the *Journal of Research*.

For stresses which were nearly the maximum stresses in the test columns the lateral deflection was very small and the behavior of all the members was satisfactory.

The useful maximum load (column yield strength) for silicon-steel columns was 1.55 times that for carbon-steel columns. For carbon-manganese-steel columns it was 1.71 times that for the carbon-steel columns. These ratios are practically the same as the ratios of the tensile yield strengths of the materials.

This investigation showed that short, sturdy columns made of the alloy structural steels which are now available will support greater loads than columns of the same size made of the carbon steel used in the past for most of our large buildings and bridges.

Most important of all, these tests gave definite assurance to the Port Authority and to the public that the silicon-steel tower columns in this bridge will carry the intended loads safely.

THERMAL DECOMPOSITION OF TALC.

Although talc has been used for many years by ceramists in the manufacture of specialties, it is only recently that its use as an ingredient of whiteware and clay refractories has become widespread. Because of the promising results obtained, it seemed desirable to have fundamental data on the process and products of its thermal decomposition.

A sample of nearly pure talc was investigated, determinations being made of the weight losses and accompanying changes in true specific gravity, and of heat effects. X-ray and microscopic examinations of the heated samples were also made. The results indicate that water in excess of one molecule, and which is not held purely by adsorption or by capillarity, may be held electrostatically between basal cleavage planes, and was driven off mostly by heating between 380° and 500° C. This loss is not accompanied by any change in crystal structure. The remaining water was driven off between 800° and 840° C., resulting in a transformation of the talc to enstatite and amorphous silica.

The inversion of the enstatite to clinoenstatite takes place gradually, being observable in material heated to 1,200° C. and complete in material heated at 1,300° C. The material heated at 1,300° C. indicates also complete conversion of the amorphous silica to cristobalite.

SAGGER CLAYS AND SAGGER BODIES.

The practice of placing unburned pottery in clay containers or saggers, as they are commonly termed, was started hundreds of years ago because of the necessity of protecting the ware from furnace gases, ashes, soot, and rapid changes in temperature during the maturing stage, and also to make possible the stacking of the ware to heights not otherwise readily attainable. Saggers are an important item in the manufacturing cost of various ceramic products, not only because of breakage and bulging of the saggers but also because of spoiled ware resulting therefrom. It was with the object of increasing the comparatively short life of the sagger and thereby decreasing the heavy losses sustained annually by the industry that the Bureau undertook a study of the properties which were considered of major importance in accomplishing this end. RP827 in the *Journal of Research* for September summarizes briefly results published in seven progress reports and also includes some unpublished data covering the effect of additions of talc to a sagger body.

Briefly, 51 clays of the type used for saggers were studied with respect to chemical composition and various physical properties both before and after heating at different temperatures. The tests indicated that certain clays had similar properties; consequently the later phases of the investigation were limited to 36 clays. Eighty-five sagger bodies were prepared from these clays by blending equal weights of each of two clays and either a fine-graded or a coarse-graded grog. Various physical properties of the bodies were determined after heating them at one of each of several temperatures within the approximate range at which saggers are used commercially, so that 139 groups of specimens of different properties were obtained and tested. Twelve commercially prepared bodies were also studied. The results obtained should enable the manufacturer to (1) produce saggers suitable for particular services, and (2) predict the probable life with respect to resistance to thermal shock if certain properties of the constituent clays are known.

THE FRANKLIN INSTITUTE

HENRY BUTLER ALLEN APPOINTED DIRECTOR OF THE MUSEUM.

The Executive Committee has appointed Mr. Henry Butler Allen Director of the Museum. He will assume his duties on October first.

Mr. Allen was born in Greenfield, Mass., in 1887; his great great grandmother, Mrs. Jerome Ripley, née Sarah Franklin, was the great granddaughter of the uncle and godfather of Dr. Benjamin Franklin. Mr. Allen was educated in the New England schools and Amherst College, and in 1911 received the degree of Metallurgical Engineer from Columbia University School of Mines. He is a member of the Delta Kappa Epsilon.

His first appointment was U. S. Examiner of Iron and Steel for the Port of New York. From 1913 to 1917 he was metallurgical engineer for Henry Disston & Sons, Inc. During the World War Mr. Allen served on the staff of the Chief of Ordnance of the American Expeditionary Force and attained the rank of captain. From 1920 to 1922 he was Vice-President of the Dodge Steel Company, later becoming Chief Metallurgical Engineer for Henry Disston & Sons, Inc., and continuing in that position to date.

He is a member of the American Institute of Mining and Metallurgical Engineers, the American Society for Testing Materials, the American Society of Metals, the American Iron and Steel Institute, the British Iron and Steel Institute, the British Institute of Metals and the Society of Automotive Engineers. He has presented communications to several of these societies and has contributed many papers to the technical press.

Mr. Allen became a member of The Franklin Institute in 1916. Because of his great interest in science and technology, he was appointed a member of its Committee on Science and the Arts in 1917. He has served on this Committee contin-

HENRY BUTLER ALLEN, DIRECTOR

uously until the present time, and during 1930 was active as its chairman. He was elected a member of the Board of Managers in 1932.

* * *

DR. HOWARD MCCLENAHAN, for ten years Secretary of the Institute, and for four years Director of its Museum, has been given a leave of absence for one year, beginning September first. The management hopes that during this time Dr. McClenahan will fully recover his health and will be able to resume his duties as Secretary of the Institute. In his absence, the Assistant Secretary and Librarian, Mr. Alfred Rigling, will carry on the work of his office.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

RESIDENT.

DR. GARTH BOERICKE, Physician, Merion, Pa.

MR. W. B. COLEMAN, Consulting Chemist and Metallurgist, W. B. Coleman and Company, Philadelphia, Pa.

MR. R. BELKNAP COLLINS, Cost Engineer, Pennsylvania Railroad, Philadelphia, Pa. For mailing: 503 Germantown Manor, Mt. Airy, Philadelphia, Pa.

MR. VERNON GODSHALL, Teaching Chemist, Temple University, Philadelphia, Pa. For mailing: 1421 Arch Street, Philadelphia, Pa.

MR. MARCEL A. LISSMAN, Consulting Engineer, Camden, New Jersey.

MR. E. R. PIORE, Junior Engineer, RCA Manufacturing Company, Camden, New Jersey. For mailing: 253 South Twenty-first Street, Philadelphia, Pa.

MR. JOHN F. SCOTT, Research Engineer, Anthracite Institute Laboratory, Primos, Delaware County, Pa.

NON-RESIDENT.

MR. RUSSELL E. MACKENZIE, Sales Manager, Carburetor Division, Detroit Lubricator Company, Detroit, Michigan. For mailing: 1199 Atkinson Avenue, Detroit, Michigan.

CAPTAIN OSCAR U. ZERK, President, Zerk Corporation, Chicago, Illinois.

STUDENT.

MR. JAMES BROOK, 2268 Hollinshed Avenue, Merchantville, New Jersey.

MR. JOHN F. GLENN, 1663 Bridge Street, Philadelphia, Pa.

MR. HARVEY L. HANSBERRY, 4059 Manayunk Avenue, Philadelphia, Pa.

MR. RICHARD D. HECKEL, 201 Elm Terrace, Narberth, Pa.

MR. THOMAS T. HILL, 1107 Spruce Street, Philadelphia, Pa.

MR. DAVID G. MORRIS, 1152 North Sixtieth Street, Philadelphia, Pa.

MR. PAUL C. ROSENBLOOM, 4843 North Franklin Street, Philadelphia, Pa.

MR. OSCAR J. SELTZER, 1520 Spruce Street, Philadelphia, Pa.
MR. RICHARD WALTERS, 124 South Sixty-second Street, Philadelphia, Pa.

MUSEUM MEMBERSHIP.

STUDENT.

MR. CLYDE AUERBACH, 1464 Wildwood Avenue, Camden, New Jersey.
MR. MARVIN BLITZSTEIN, 4332 North Fifteenth Street, Philadelphia, Pa.
MR. CLYDE CARSON, 1621 West Erie Avenue, Philadelphia, Pa.
MISS HILDA MELINSON, 5720 Wyndale Avenue, Philadelphia, Pa.
MR. LEON PINSKER, 4251 Leidy Avenue, Philadelphia, Pa.
MR. WALTER ROWLAND, 126 Ashby Road, Upper Darby, Pa.
MR. RICHARD S. STERN, Rose Tree Road, Media, Pa.
MR. JOHN S. STEVENSON, 603 Locust Avenue, Philadelphia, Pa.

CHANGES OF ADDRESS.

MR. B. F. BARDO, 27 Upland Road, Quincy, Massachusetts.
MR. WILLIAM H. BARTON, JR., American Museum of Natural History, Seventy-seventh Street and Central Park, West, New York City.
DR. T. K. CLEVELAND, Philadelphia Quartz Company of California, Sixth and Grayson Streets, Berkeley, California.
MR. HOWARD EMMONS, 42 Prentiss Street, Cambridge, Massachusetts.
DR. CLARENCE ERROL FERREE, 2609 Poplar Drive, Baltimore, Maryland.
MR. MALCOLM LLOYD HAYWARD, Brook Road, Wayne, Pa.
MR. IRWIN S. HOFFER, Willow Avenue, Ambler, Pa.
DR. S. S. KISTLER, The Norton Company, Worcester, Massachusetts.
MR. FRANK N. KNEAS, Architects Building, Seventeenth and Sansom Streets, Philadelphia, Pa.
MR. LAWRENCE LE PAGE, Day and Zimmermann, Inc., 620 Packard Building, Philadelphia, Pa.
DR. EDWARD E. MARBAKER, Vice-President, Mellon Institute, Oakland Station, Pittsburgh, Pa.
MR. WILLIAM STANTON MONROE, Sargent and Lundy, Inc., 140 South Dearborn Street, Section 3, Old Post Office Annex, Chicago, Illinois.
MR. RICHARD MAXWELL PEARL, 12755 Evenston Avenue, Detroit, Michigan.
DR. SAMUEL E. POND, Box 224, Falmouth, Massachusetts.
DR. CHARLES R. RICHARDS, 1725 Summit Avenue, Minneapolis, Minnesota.
MR. WALTER SCHULZ, 710 South Street, Elizabeth, New Jersey.
MR. SAMUEL SPITZ, 330 North Detroit Street, Los Angeles, California.
MR. JAN A. VILJOEN, 531 Dudley Avenue, Narberth, Pa.
MR. FRANCIS R. WADLEIGH, 806 Prince Street, Alexandria, Virginia.
MR. CHARLES LARRICK, Y.M.C.A., 1421 Arch Street, Philadelphia, Pa.
MR. ALFRED WOLF, 2607 Boston Place, Tulsa, Oklahoma.

NECROLOGY.

Mrs. George L. Estabrook, Philadelphia, Pa.
Mr. I. H. Francis, Philadelphia, Pa.
Mr. A. Saunders Morris, Philadelphia, Pa.

LIBRARY NOTES.

RECENT ADDITIONS.

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BOOK REVIEWS.

MEN, MIRRORS, AND STARS, by G. Edward Pendray, 339 pages, illustrations, plates, 14 × 21 cms. New York and London, Funk and Wagnalls Company, 1935. Price $3.00.

Numerous popular books on astronomy have been issued in the last few years, but, in practically all, descriptions of the instruments that have made modern celestial research possible are rather incidental. Here is a book that gives a complete story of how the telescope came into being, how it was developed into the great instruments of modern observatories, and what may come in the future. Important accessories to the telescope, the spectroscope, photometer, interferometer, thermocouple, etc., are also described. And all through the author tells how these instrumental advances have increased our knowledge of the universe.

Considering the wide field Mr. Pendray has covered, it is done extraordinarily well, the reviewer has noted only a few, very trivial errors, such as crediting the invention of the heliometer to Fraunhofer instead of to Servington Savery, and stating that the 18-inch refractor of the Flower Observatory of the University of Pennsylvania was built by the John A. Brashear Co., instead of Warner and Swasey.

In his descriptions of modern observatories, the author has performed a useful labor in including in one place the accounts of these institutions in the printed literature that they have issued. But it must not be thought that the book is a compilation, for the greater part is entirely original with information that Mr. Pendray has gathered after what must have been extensive research. The appendix includes a list of important observatories of the United States (which omits the Sproul Observatory of Swarthmore College, though it does include the Flower Observatory) and Canada, with important facts concerning them, such as equipment, staff and chief fields of work. A few particularly outstanding foreign observatories are included, and there is a list of the great telescopes of the world, including refractors 24 inches or larger (though again omitting the 24-inch Sproul Observatory instrument) and reflectors 36 inches or larger in aperture.

The author's style of writing is clear and interesting, and the book can be heartily recommended to anyone who wishes to know more of the methods used by the astronomer, and the steps that have led to them.

JAMES STOKLEY.

AN INTRODUCTION TO ASTRONOMY, by Robert H. Baker, Ph.D. 312 pages, illustrations, tables, 15.5 × 23.5 cms. New York, D. Van Nostrand Company, Inc., 1935. Price $3.00.

Several books on astronomy from Dr. Baker's pen have appeared during the last few years, and this is fully up to the high standard set by the previous ones. It is intended as a text for a short introductory course in astronomy, and the student who is fortunate enough to use it should secure an excellent general knowledge of this fascinating science. The style is not at all that of a typical textbook, so it should prove of value also to the general reader who wants a complete, well-written and authoritative as well as interesting book on astronomy.

Starting with the changes that can be observed in the sky by anyone, and the motions of the Earth that causes them, he immediately proceeds to introduce his reader to the stars and constellations, a part which many text-book authors are inclined to neglect. True, knowledge of the constellations is not essential to an astronomer, but those who have the broadest acquaintance with the subject know them well. Certainly a person who is supposed to be well-educated should be familiar with the most conspicuous ones, and it is gratifying to find Dr. Baker giving them such a prominent place.

In succeeding chapters, he considers the Moon, the planets, the Sun and its light, the stars, nebulæ, the Milky Way system, and the others beyond. Numerous photographs, some from negatives made with the greatest telescopes, show many of the things described. Members of The Franklin Institute will be particularly interested in the excellent photograph of the Fels Planetarium on page 78. In addition cleverly drawn diagrams and star maps greatly enhance the value of the book to the student, as well as to the general reader.

This reviewer is fully aware of the difficulty encountered by the author of an elementary book on astronomy in deciding what to omit, and Dr. Baker seems, on the whole, to have chosen wisely. However, it is rather surprising to find no mention of theories of planetary evolution. Prof. Baker has given an exceedingly clear account of them in his larger "Astronomy," and one might wish that he had included the same material, condensed, in the present work. Surely any student who has had even an elementary college course in astronomy should have some idea of the Nebular Hypothesis and the place that it has occupied in the history of scientific thought. He should know why it has proven untenable, and should have a speaking acquaintance with the Planetesimal and Tidal Theories, the principal ones that have replaced it.

Some minor errors are to be found in the map on page 232 of the coming total solar eclipses seen in North America. This has evidently been redrawn from the charts in Oppolzer's "Canon der Finsternisse" (Vienna, 1887), the standard authority for data on past and future eclipses. Accurate as Oppolzer's tables are, the charts only purport to be approximate, and frequently differ from the real path by many miles. Thus, Oppolzer, and Baker, show the path of totality of 1963 July 20 as missing the United States entirely, whereas it will cross central Maine. Similarly, the 1970 March 7 eclipse track will cross northern Florida and the coast line of Georgia, South and North Carolina, passing to sea north of Cape Hatteras, and touching land again in Nova Scotia, though Oppolzer's map shows it crossing central Florida, and then going to sea. A more accurate map of the eclipse tracks in the United States from 1933 to 2000 has been prepared by Dr. C. H. Smiley, of Brown University, and published in *Popular Astronomy*, Vol. XL, page 526 (November, 1932).

But this is not an error that would confuse the reader, and in the important things the book deserves the reviewer's highest praise. It is entirely up to date, and the scientific accuracy is assured not only by Prof. Baker's own high standing but by the note in the preface that Prof. Frank Schlesinger, of Yale, has given the entire manuscript a critical reading.

JAMES STOKLEY.

COURS PRATIQUE D'ÉLECTRICITÉ INDUSTRIELLE A L'USAGE DES ÉCOLES PRATIQUES DE COMMERCE ET D'INDUSTRIE, DES ÉCOLES PROFESSIONNELLES, DES ÉCOLES PRIMAIRES SUPÉRIEURES, par MM. Léon Bouthillon, Ingénieur en chef des Télégraphes, Répétiteur à l'École Polytechnique, et M. Goudonnet, Directeur de l'École pratique de Radioélectricité, Premier volume, Les lois fondamentales de l'Électricité. 305 pages, illustrations, 12 × 19 cms. Paris, Gauthier-Villars, 1934. Price 25 frs.

This little paper-back volume represents the first of three texts that are to constitute a practical course in industrial electricity. The complete course is broken down into eight parts, the first two of which are included in the volume under consideration, viz.: "Part One—The fundamental laws of electricity," and "Part Two—The Alternating Current."

The whole subject is presented in a clear and easily understood manner. Many illustrations amplify the written word and the various relationships are couched in that universal short-hand—the algebraic equation. Published in the French language, this book should be of double value to students taking electrical courses. They not only will have an opportunity to acquaint themselves better with the scientific aspects of the language but simultaneously will profit by a review of the principles of electricity.

T. K. CLEVELAND.

THERMOCHIMIE. By W. Swietoslawski, Professor of Physical Chemistry at the Polytechnical School of Warsaw. Translated by M. Thon. 379 pages, 15.5 × 23 cm. Paris, Librairie Felix Alcan, 1933. Price 60 francs.

This monograph is a translation into the French of Swietolawski's Thermochimie (Volume VIII, Handbuch der allgemeinen Chemie, 1927) brought up to date (1933) and supplemented by a translation of Volume III of Manuel de Chimie physique from the Polish.

We have here a masterly treatment of the whole range of thermochemical measurements from the pen of one who has contributed extensively to the present amazing refinement of measurement of heats of reaction. An experimentalist and critic of the highest order, he has contributed greatly to the establishment of thermochemical standards. His critical evaluation of data from earlier thermochemists has been brought to the point of working out factors by which much of the earlier work may be corrected, a remarkable achievement. Thus the thermochemical values of Berthelot (first period) may be corrected by the factor 0.9900, and for the same author and collaborators (second period) by the factor 0.9965. A few other factors selected below will serve to indicate that the degree of precision attainable in thermochemical measurements is equal to the precision of most analytical and many physical measurements. Values given by Louguinine may be multiplied by 0.9970, Fischer and Wrede (1908 and 1910), by the same figure, Richards (1910), 0.9970, Richards (1918), 0.9994, Swarts, 0.9981.

In Part One the author differentiates between calorimetric measurements and thermochemical measurements. The latter are restricted to cases where chemical reaction takes place; the former comprise a variety of measurements such as specific heats, heats of solution, heat of fusion, etc. Thermochemical measurements involve usually very considerable added difficulties due to time effects. Methods and apparatus are fully discussed, with the aid of numerous drawings.

Part Two consists of chapters devoted to analyses of data published for various classes of compounds. Representative chapters deal with hydrocarbons, alcohols, acids, amines, nitriles and isonitriles and halogen compounds.

Part Three deals with the thermochemistry of atomic linkages, with special chapters devoted to the thermochemistry of electrolytes and ionization potentials.

Part Four entitled "Chemical Affinity" is concerned with the thermodynamical aspects of heat measurements. An interesting review of the historical controversy on the chemical affinity principle of Berthelot is given.

The volume throughout is remarkable for completeness and critical effort. Thermochemistry is a subject of more than 100 years of intensive experimental effort. The bibliographical labor alone represents a large assignment.

The technique of this work is exacting but the details have been worked out to a point where measurements can often be made quickly and with high precision. This technique sometimes can be applied where all others fail. It is a subject of the widest and most fundamental importance.

LESLIE R. BACON.

THE SOCIOLOGY OF INVENTION, AN ESSAY IN THE SOCIAL CAUSES OF TECHNIC INVENTION AND SOME OF ITS SOCIAL RESULTS, by S. C. Gilfillan, Ph.D. 185 pages, diagrams, 14 × 20.5 cms. Chicago, Follett Publishing Company, 1935. Price $2.00.

The book is a formal study primarily on the social causes and results of invention. Causes and results are considered together because, as is stated, inventions are the result of social and other forces and become themselves the cause of further inventions, or of obstruction to invention. It contains a wealth of information for the analyst.

The author's method of constructing his treatment is noteworthy and unique. He first outlines thirty-eight social principles of invention, conveniently divided into topics such as those pertaining to the nature of invention; changes evoking invention; factors fostering, retarding and locating invention; effects of invention, etc. Then in following chapters he discusses these in an illuminating way bringing forth factual data of sometimes surprising nature. Throughout his discussion he refers to a certain category of inventions, namely the ship, to demonstrate his inductions. In the history of ship evolution, designers have been particularly averse to revolutionary innovations, because of the danger to lives and much property, but this has probably been weakened in the modern age with its growth of safety, its stirring together of intelligences, and subjecting to standard customs of science all directive and engineering men of whatever industry. As one reads the book there is an increasing tendency to approve the author's selection of the ship as an illustration, rather than other more difficult references, because of its ease of understanding.

A particularly interesting part of the book is that devoted to the definition of the word "invention." It is stated that of all the words in the language this is perhaps the hardest to define. Many competent people define inventions on a ship in various ways and the Supreme Court has said that the word cannot be defined in such a manner as to afford substantial aid in determining whether a particular device involves an exercise of the inventive faculty or not.

In another part of the book it is shown that the impression given by many

writers that inventors are swarming more and more to the Patent Office is not true. Since 1885 American patents have been increasing only in proportion to the population, no faster, despite all the incitements from increasing industrialism and civilization. We were granting a patent yearly for each 2675 inhabitants in 1883–7, and 1 per 2691 in 1925–9, since when the depression has reduced applications by a third.

Throughout the book citational notes are made and are listed in the back and as footnotes. Bibliographies on social causation, social effects, psychology of invention, inventive method, and histories of invention are appended as is a comprehensive index.

The book can be recommended to any serious minded reader who wishes to learn more about that desire, common to most of us, to produce something new and useful in the world.

R. H. OPPERMANN.

OCULAR DIOPTRICS AND LENSES, by G. F. Alexander, M.B., 216 pages, illustrations, 16.5 × 23 cms. Baltimore, William Wood and Company, 1934. Price $4.75.

This work was originally intended as an article on static refraction and magnification but the author states that for its elucidation he was compelled to make one incursion after another into other fields of optics which finally led to this book. The book is an authoritative treatise. It consists of twenty-one chapters beginning with statements on the laws of refraction and simple and compound refracting systems, from which it goes into lenses—concave, convex and a very good treatment on the crystalline lens and static and dynamic refraction of the eye. The student is then taken through chapters on the correction of ametropia, regular astigmatism, and the effect of the forward movement from the eye on lenses on the static and dynamic refraction, and on the apparent size of objects. A final chapter is devoted to the deviation of light by prisms and decentered lenses.

The treatment is almost entirely mathematical and use is made of diagrams whenever it is necessary to make the work easier to understand. A comprehensive index completes the work.

Those concerned with the art of ocular refraction will find in this book a thorough study on subjects that receive little notice in text books and teaching and that are difficult to explain.

R. H. OPPERMANN.

ELEKTROTECHNIK EIN LEITFADEN FÜR STUDIUM UND PRAXIS, Herausgegeben von Dr.-Ing. G. Bolz, Dr.-Ing. F. Moeller, Dipl.-Ing. Th. Werr. Band II, Teil 3 und 4, Gleich- und Wechselstrommaschinen, von Dr.-Ing. F. Moeller und Dipl.-Ing. Th. Werr, 125 pages, illustrations, 16 × 24 cms. Leipzig und Berlin, B. G. Teubner, 1935. Price 10 marks.

In the June number of the JOURNAL OF THE FRANKLIN INSTITUTE on pages 764–5 there was reviewed Volume I of a work on electrical engineering treating with the fundamentals of electricity and magnetism to prepare the general engineer for the study of Volume II. The present book (Volume II) is the continuation of the work and deals with direct and alternating current machinery.

This is, like the previous book, divided into two parts, the first covering the generation of voltage and construction of direct current machines. Description is given on the operation and factors affecting the efficiencies of such machines.

The second part deals with alternating currents, beginning with transformers with the theory of induction and proceeding then to the induction motor, followed by the synchronous machine. A special section is devoted to commutator machines, including single and polyphase motors and finally a brief treatment is given the rectifier and the possibilities of its use in the distribution net.

The book, like its predecessor is outstanding for its simple clarity. About the same prerequisites regarding mathematics are necessary, a very slight knowledge of the calculus. Trigonometry is employed through the use of vectors. Profusely illustrated with photographs, diagrams and drawings, it can be recommended as an excellent compact text.

R. H. OPPERMANN.

ENERGIE- ÜBERTRAGUNG UND UMWANDLUNG MIT WECHSELSTROM, EINHEITLICHE THEORIE DER LEITUNGEN TRANSFORMATOREN UND MASCHINEN, von Dr.-Ing. Paul Werners. 204 pages, illustrations, 16 × 24 cms. Leipzig und Berlin, B. G. Teubner, 1935. Price 18 marks.

A thorough and vigorous treatment of energy transfer and conversion with alternating current in a handy compact volume. It is exceptional in its completeness and brevity.

At no expense in influencing fundamental principles, the electric machine is viewed from the standpoint of a machine for the transfer of energy, and its conversion from one kind to another. Thus, an induction motor is likened to a transformer, transferring electrical energy from the stator to the rotor and to mechanical energy. It is claimed that by making this view the basis of treatment, limitation of space is accomplished as well as efficiency in grasping the subject. The use of symbols is made in mathematical calculations, similar to that of Steinmetz in his books on Alternating Currents.

The book has twelve chapters. It begins with a brief chapter on losses and then there is taken up mathematical resources upon which the rest of the book is founded. Next, the student progresses through chapters dealing with single and polyphase currents, the stationary and rotating field transformer, and the induction and synchronous motor. It is well illustrated with diagrams and contains a comprehensive index.

While the book only contains some 200 pages, it has all the information necessary for a thorough understanding of the subjects treated.

R. H. OPPERMANN.

MECHANICAL AND ELECTRICAL EQUIPMENT FOR BUILDINGS, by Charles Merrick Gay, A.B., B.S., and Charles De van Fawcett, E.E. 429 pages, illustrations, tables, 15 × 23.5 cms. New York, John Wiley & Sons, Inc. London, Chapman & Hall, Ltd. 1935. Price $5.00.

As can be seen from the title, this book is not devoted to a basic subject, rather it is a consideration of the application of a number of basic subjects to achieve an end. The end to be achieved is the servicing and the supply of services

to buildings. In this, the treatment satisfies a very special need, by including between its covers essentials embracing simple concise statements of the fundamental theories involved together with their broad applications in architectural design that would be difficult to find between the covers of any other one book.

The book is divided into five sections. The first and second cover chapters on water supply, plumbing and drainage, and include design of equipment as well as descriptions of supply and estimated requirements. Section three is devoted to heating and air conditioning. Heating is first discussed by reference to heat and heat losses, then warm air, steam and hot water heating are taken up. A separate chapter considers radiators, piping, boilers and fuel. Fifty-two pages are devoted to air conditioning, theory, design, application and equipment.

Section four on electrical equipment for buildings is especially interesting from a practical point of view in that it gives a very clear treatment on wiring systems and maintenance among other subjects including general electrical machinery, elevators and lighting both interior and floodlighting. A new field of activity in building design is taken up in section five, that of acoustics. In this, the last section of the book twenty-five pages discuss and describe reverberation, noise and noise levels, insulation of sound, standards for design, etc.

Just enough treatment is given in this book to give the architectural student and the practicing architect an acquaintance with basic principles. In addition to this the book should be quite valuable to building engineers as a reference book for the more theoretical and basic part of their work as well as giving a brief up-to-date treatment on the newer subjects involved in their work.

R. H. OPPERMANN.

RADIO-ÉLECTRICITÉ GÉNÉRALE, TOME I, ÉTUDE DES CIRCUITS ET DE LA PROPAGATION, par R. Mesny. 375 pages, tables, illustrations, 16.5 × 25.5 cms. Paris, Etienne Chiron, Price fifty francs.

The title "General Radio-Electricity," is applied to an ambitious undertaking consisting of three volumes. Volume one, herein under consideration, deals with a study of various types of circuits and the propagation of electromagnetic waves. Volume two, apparently to appear later, covers the subject of transmitters and receivers, while the third unit deals with the subject of directional radio.

Arrangement of the material follows the conventional order of most textbooks. The first chapter reviews the properties of differential equations, the geometrical representation of complex quantities and characteristics of hyperbolic curves. Damped and undamped oscillations in coupled circuits are treated separately. Resonance and selectivity are accorded a graphical treatment. A chapter of self-induced oscillations introduces a discussion of the three-element electronic tube and this is followed by the matter of synchronized oscillatory circuits. A chapter on open circuits precedes a practical discussion of oscillations along wires and on antennæ. Energy losses in resistance bobbins, in dielectrics and by hysteresis in iron cores command considerable attention. The book is terminated by a discussion of wave propagation in space and along the ground.

Written in French, the volume should be of double value to the reader. It is not only a source of information on electrical waves but also a means of improving one's ability to read scientific French papers.

T. K. CLEVELAND.

MATHEMATICS AND THE QUESTION OF COSMIC MIND, WITH OTHER ESSAYS, by Cassius Jackson Keyser, Adrain Professor Emeritus of Mathematics, Columbia University. 121 pages, 13 × 19.5 cms. New York, Scripta Mathematica, 1935.

This little book is made up of reprints with some alterations, of essays from *The American Scholar*, the *Scripta Mathematica* and the *Yale Law Journal*. It is a part of the Scripta Mathematica Library.

The title essay has been given the third place because it cannot be rightly understood before reading the articles preceding it. While these three constitute a logical sequence, the remaining essays may be taken up in any order.

The book is a good one for serious-minded readers to read of an evening when it is desired to pass the time in an instructive and informative way. Extensive mathematical knowledge or a high degree of mathematical aptitude are not necessary. The first essay, "The Meaning of Mathematics" points out the folly of obtaining information regarding the essential nature of mathematics from a standard dictionary and presents cleverly an analysis of the coveted insight. We often hear of, and some of us can appreciate a mathematical mind when we encounter one. Few have taken the time and in fact are capable of defining just what this is. In the discussion the meaning is brought out clearly by contemplating mathematics as a certain kind of intellectual enterprise characterized by its aim, by contemplating it as a body of achievements or of knowledge characterized by the type of propositions constituting the body, and by contemplating it as a certain type of thought or way of thinking characterized by procedure and method.

The second article is entitled "The Bearings of Mathematics." The aim is to give a fair sense of the bearings of mathematics as distinguished, on the one hand, from mathematics itself, and, on the other, from mathematical applications. It is stressed that these three must not be confused in order that the third essay be read understandingly.

"Mathematics and the Question of Cosmic Mind" treats on the question: What evidence, if any, does mathematics furnish to support the thesis that our universe is, essentially and fundamentally, a realm of mind? This third essay, when read, furnishes one with food for much thought. Based on a clarification of the essential meanings of the major terms involved, a positive affirmative answer is reached.

The three remaining essays are equally as interesting. "Mitigating the Tragedy of Our Modern Culture" and "On the Study of Legal Science"—both are well worth reading. The last essay, on "William Benjamin Smith," is a brief and well written biography which illustrates admirably the mind of a great man—a mathematician, physicist, poet, philosopher, historian, teacher and critic.

R. H. OPPERMANN

COMMÉMORATION DU CENTENAIRE DE LA MORT DE JOSEPH-NICÉPHORE NIÉPCE, INVENTEUR DE LA PHOTOGRAPHIE. MANIFESTATIONS ORGANISÉES À CHALON-SUR-SAÔNE EN JUIN 1933. 78 pages, illustrations, 16 × 24 cms. Paris, Société Francaise de Photographie et de Cinématographie.

The celebration of the centenary of the demise of the celebrated inventor, who first succeeded in devising a workable method for the production of pictures

from the image of the camera obscura, provides an opportunity for securing an accurate historical record of the beginnings of photography. These ceremonies in their social, nationalistic and technical aspect are described in detail in this volume.

There are many illustrations of apparatus used by Niépce and reproductions of prints which were made by him; and not the least interesting and informative are the several addresses made by noted participants in the ceremonies. Among these, that of M. Potonniée, who is the author of a history on the discovery of photography. These portions of the document especially are of particular value in establishing the merits of Niépce in evolving a process by which permanent pictures were actually made by him. In that connection, it is of interest to note an observation of a British correspondent with the Committee: "I may mention that the bitumen method of 'printing' on metal was in use in London until recently."

L. E. P.

THE THEORY OF ATOMIC SPECTRA, by E. U. Condon, Ph.D., and G. H. Shortley, Ph.D. 441 pages, illustrations, 17.5 × 27 cms. New York, The Macmillan Company; Cambridge, University Press, 1935. Price $11.00.

A survey of the present status of the problem of interpreting the line spectra due to atoms, this book begins in its introductory chapter with a brief historic review, then, in order to make the subject matter more useful for independent reading the reader is given a brief account of the principles of quantum mechanics. Here reference is made to the work of Dirac in the formulation of the laws of quantum mechanics, as this is most suited to the purposes of the book. Following this, in the same chapter, there is discussed matrix mechanics, on which subject, the authors state, it has been too little recognized that a matrix is not fully useful in the transformation theory unless the relative phases of the states to which the components refer are in some way specified. The phase choice is arbitrary but one choice does have to be made and adhered to throughout a given set of calculations. After this discussion is centered on the perturbation theory.

The succeeding two chapters (III and IV) are devoted to the presentation of special results from the general theory which are used throughout the book. In Chapter III under the heading of "Angular Momentum," the properties of the dynamic variables analogous to the components of angular momentum in classical dynamics, and of the electron spin are developed. These have constant application throughout the theory of atomic spectra. "The Theory of Radiation" which is the title of Chapter IV, contains the general theory underlying the radiative process.

The detailed development of the theory of atomic spectra begins with Chapter V on one electron spectra. Hydrogen-like spectra are treated on followed by the alkali spectra, foreseeing for the alkali spectra the justification of the assumption that these may to a good approximation be regarded as one electron spectra. The remaining thirteen chapters of the book are devoted to a continuing unified logical deduction of the structure of the spectra of atoms. The theory is given in full detail, the results amply documented with comparisons with experimental facts. They include such subject headings as the Russel-Saunders case—energy levels—eigenfunctions—line strengths, treatments on coupling, the transformations in the theory of complex spectra as well as the Zeeman effect and Stark effect.

The authors state that while the interpreting of the line spectra due to atoms may be in a fairly closed and satisfactory state, the period of fruitful research in atomic spectra is not necessarily at an end. Readers of this book may see many places where more experimental information and better or more detailed theoretical calculations are desirable. Also fundamental questions are still outstanding in regard to the relativistic theory of the many-electron problem and also in regard to the theory of the interaction of radiation and matter.

R. H. OPPERMANN.

PHYSICS, by Erich Hausmann, E.E., Sc.D., and Edgar P. Slack, S.B., 776 pages, illustrations, 15 × 22.5 cms. New York, D. Van Nostrand Company, Inc., 1935. Price $4.00.

An unusually clear and readily adaptable treatise on mechanics, heat, electricity and magnetism, sound, and light. The book throughout gives a gradual and logical approach to each subject from the simple to the more complex, to the end that it is more interestingly presented and comprehensive.

In all there are forty chapters well divided into the five subjects. Mechanics and heat cover 363 pages, and provide a thorough grounding in these subjects. Illustrative of the completeness of treatment can be seen from the very beginning where, in discussing the addition of lengths, analytical solutions instead of actual measurements are suggested. Here in three pages is given the gist of the trigonometry of triangles necessary to accomplish this object.

Electricity and magnetism is dealt with from the fundamentals of the electric circuit through parts on batteries and direct current machines to alternating currents, thermionics and the fundamentals of electrical communication. The chapters on light and sound contain all of the basic physics on these subjects.

As a whole, the book is representative of a modern text. Modern viewpoints are used to present the subject matter in such a way that its absorption is facilitated. In presenting derivations of physical laws and relationships, the purpose of the proof and the course to be followed are stated before embarking upon the detailed description and mathematical processes. A list of problems appears at the end of each chapter for exercises. The answers appear at the end of the book.

For the essentials of physics, anyone with part high school training or better can find no better text.

R. H. OPPERMANN.

THE OPTICAL BASIS OF THE THEORY OF VALENCY, by R. de L. Kronig, Ph.D., University of Groningen. 246 pages, illustrations, 14.5 × 22.5 cms. New York, The Macmillan Company; Cambridge, University Press, 1935. Price $4.50.

There is presented in this book present knowledge of chemical valence derived from the study of optical phenomena. It is a picture of the progress achieved by the application of purely physical methods.

The book has six chapters the first of which is introductory dealing briefly with facts of chemistry, atomic theory and wave mechanics, and atomic models and their limitations. Next, attention is focussed upon the geometrical aspects of the structure of atoms by means of x- and cathode rays, and following this, in a chapter on atomic spectra and the periodic system, there is treated the question

of binding forces and binding energies governing this structure. This chapter restricts itself to the case of the isolated atom. The program adheres to the questions of finding a suitable theoretical basis for subdividing the various electrons in the atom into groups, investigating the way in which the energy necessary for the removal of an electron from the atom depends upon the group to which it belongs as well as upon the atomic number, and finally how far the classification of the individual electrons suffices to characterize the state of the atom as a whole and in case this classification proves inadequate, which additional features may serve to specify the atomic state uniquely.

In the chapter on Band Spectra and Chemical Binding in Diatomic Molecules there is investigated the interaction of two atoms and it shows that investigations of the chemical binding fall into two parts: viz. on the one hand the collection from band spectra of numerical data, characteristic for the different molecules, such as the equilibrium distances, the vibration frequencies, the dissociation energies of the different electronic states, and more especially of the ground state; on the other hand the interpretation on the basis of wave mechanics, of the energy curves, derived from these data, and their relation to the concepts of chemistry. This same division of labor is adhered to in the chapter on polyatomic molecules with some differences.

The last chapter of the book considers the various kinds of optical dissociation processes and the phenomena of thermal dissociation in the light of the information regarding molecular structure obtained in the chapters on band spectra.

The book is well illustrated with diagrams and curves, and contains a number of good photographic plates which not only make for easier understanding but are very interesting in themselves. At the end of each chapter a selected bibliography is appended. There is included at the back of the book a subject and name index, and a table of electron configurations. All greatly increase the value of the book.

R. H. OPPERMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Report No. 507, Tests of Nacelle-Propeller Combinations in Various Positions with Reference to Wings. VI—Wings and Nacelles with Pusher Propeller, by Donald H. Wood and Carlton Bioletti. 31 pages, tables, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1934. Price ten cents.

This report is the sixth of a series giving the results obtained in the N.A.C.A. 20-foot wind tunnel on the interference drag and propulsive efficiency of nacelle-propeller-wing combinations. The first three reports of the series gave the results of tests of radial-engine nacelles with tractor propellers and numerous types of engine cowling. Tests were made with the nacelles in various positions with respect to a thick monoplane wing and a Clark Y monoplane wing. The fourth report covered tests of tandem-propeller nacelles for radial engines with numerous types of cowling, in various positions with reference to a thick monoplane wing. The fifth report of the series gave the results of tests of an N.A.C.A. cowled nacelle with tractor propeller in various positions with reference to a biplane wing cellule of Clark Y section. The present report gives the results of tests of a radial-engine nacelle with pusher propeller in 17 positions with reference to a

Clark Y wing; tests of the same nacelle and propeller in three positions with reference to a thick wing; and tests of a body and pusher propeller with the thick wing, simulating the case of a propeller driven by an extension shaft from an engine within the wing. Some preliminary tests were made on pusher nacelles alone.

The Clark Y wing had a 38-inch chord and a 15-foot 10-inch span. The thick wing had a 5-foot chord, a 15-foot span, and a thickness of 20 percent of the chord. The nacelle was built around a 4/9-scale model of a Wright J-5 radial air-cooled engine and was fitted with a cowling of the variable-angle ring type. The body simulating the extension-shaft case was formed by fairing into a thick wing the electric motor used for driving the propeller. The propeller was a 4-foot-diameter model of the Navy No. 4412 adjustable metal propeller.

Report No. 519, Spinning Characteristics of Wings. I—Rectangular Clark Y Monoplane Wing, by M. J. Bamber and C. H. Zimmerman. 20 pages, diagrams, 23 × 29 cms. Washington, Superintendent of Documents, 1935. Price ten cents.

A series of wind-tunnel tests of a rectangular Clark Y wing was made with the N.A.C.A. spinning balance as part of a general program of research on airplane spinning. All six components of the aerodynamic force and moment were measured throughout the range of angles of attack, angles of sideslip, and values of $\Omega b/2V$ likely to be attained by a spinning airplane; the results were reduced to coefficient form.

The latter part of the report contains an analysis illustrating the application of data from the spinning balance to an estimation of the angle of sideslip necessary for spinning equilibrium at any angle of attack. The analysis also shows the amount of yawing moment that must be supplied by the fuselage, tail, and interference effects in a steady spin. The effects of variation of such factors as mass distribution, attitude, wing loadings, etc., upon the likelihood of a monoplane with a rectangular Clark Y wing attaining a steady spin as revealed by the analysis are considered in a discussion.

It is concluded that a conventional monoplane with a rectangular Clark Y wing can be made to attain spinning equilibrium throughout a wide range of angles of attack but that provision of a yawing-moment coefficient of -0.02 (i.e. against the spin) by the tail, fuselage, and interferences will insure against attainment of equilibrium in a steady spin.

Report No. 521, An Analysis of Longitudinal Stability in Power-Off Flight with Charts for Use in Design, by Charles H. Zimmerman. 31 pages, tables, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1935. Price ten cents.

This report presents a discussion of longitudinal stability in gliding flight together with a series of charts with which the stability characteristics of any airplane may be readily estimated.

The first portion of this report is intended, primarily, for students of the subject. The relationships governing stability characteristics are derived from equations of equilibrium referred to moving axes that are tangent and perpendicular to the instantaneous flight path. It is shown that instabilityof the motion

can arise only through an increase of linear and angular momentum in the system during one complete cycle. The interaction of events leading to increase or decrease of momentum during a cycle is explained in detail. The construction of charts showing the effects of the nondimensional parameters C_L, C_D, $dC_L/d\alpha$, $dC_D/d\alpha$, $-m_q$, and $-\mu m_\alpha$ upon the stability characteristics is explained and the effects of the more important of the aerodynamic and mass characteristics of the airplane, as revealed by the charts, are discussed.

The latter portion of the paper is devoted to a series of 40 related charts with which the dynamic stability of any airplane in power-off flight may be readily estimated. The use of the charts is explained in detail so that reference to the earlier discussion is unnecessary.

Report No. 525, Some Effects of Injection Advance Angle, Engine-Jacket Temperature, and Speed on Combustion in a Compression-Ignition Engine, by A. M. Rothrock and C. D. Waldron. 15 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1935. Price ten cents.

An optical indicator and a high-speed motion-picture camera capable of operating at the rate of 2,000 frames per second were used to record simultaneously the pressure development and the flame formation in the combustion chamber of the N.A.C.A. combustion apparatus. Tests were made at engine speeds of 570 and 1,500 r.p.m. The engine-jacket temperature was varied from 100° to 300° F. and the injection advance angle from 13° after top center to 120° before top center.

The results show that the course of the combustion is largely controlled by the temperature and pressure of the air in the chamber from the time the fuel is injected until the time at which combustion starts and by the ignition lag. The motion pictures of the combustion show that in a quiescent combustion chamber with a short ignition lag the flame first appears in the spray envelope and from there spreads throughout the chamber and that with a long ignition lag the flame may first appear in any part of the chamber. The conclusion is presented that in a compression-ignition engine with a quiescent combustion chamber the ignition lag should be the longest that can be used without excessive rates of pressure rise; any further shortening of the ignition lag decreases the effective combustion of the engine.

Report No. 526, Noise from Two-Blade Propellers, by E. Z. Stowell and A. F. Deming. 9 pages, tables, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1935. Price five cents.

The two-blade propeller, one of the most powerful sources of sound known, has been studied with the view of obtaining fundamental information concerning the noise emission. In order to eliminate engine noise, the propeller was mounted on an electric motor. A microphone was used to pick up the sound whose characteristics were studied electrically. The distribution of noise throughout the frequency range, as well as the spatial distribution about the propeller, was studied. The results are given in the form of polar diagrams.

The mechanical power radiated in the form of sound was measured for three different pitch settings of the propeller. It was found that the percentage of the

acoustical power going into the fundamental note (the "roar") became very large as the power supplied to the propeller was increased.

The effect of such sounds upon the ear was investigated both theoretically and experimentally. Computations of loudness level about the propeller at five distances were made. Attempts to check these computations experimentally showed discrepancies; explanations are given for the direction and magnitude of the deviations from the calculated loudness levels.

An appendix of common acoustical terms is included.

Report No. 527, Air Flow in a Separating Laminar Boundary Layer, by G. B. Schubauer. 12 pages, diagrams, 23 × 29 cms. Washington, Superintendent of Documents, 1935. Price five cents.

The speed distribution in a laminar boundary layer on the surface of an elliptic cylinder, of major and minor axes 11.78 and 3.98 inches, respectively, has been determined by means of a hot-wire anemometer. The direction of the impinging air stream was parallel to the major axis. Special attention was given to the speed distribution in the region of separation and to the exact location of the point of separation. An approximate method, developed by K. Pohlhausen for computing the speed distribution, the thickness of the layer, and the point of separation, is described in detail; and speed-distribution curves calculated by this method are presented for comparison with experiment. Good agreement is obtained along the forward part of the cylinder, but Pohlhausen's method fails shortly before the separation point is reached and consequently cannot be used to locate this point.

The work was carried out at the National Bureau of Standards with the coöperation and financial assistance of the National Advisory Committee for Aeronautics.

Report No. 531, The Effect of Water Vapor on Flame Velocity in Equivalent CO–O_2 Mixtures, by Ernest F. Fiock and H. Kendall King. 6 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1935. Price five cents.

This investigation, carried out at the National Bureau of Standards at the request and with the financial assistance of the National Advisory Committee for Aeronautics is a quantitative study of the effect of water vapor upon the spatial speed of flame in equivalent mixtures of CO and O_2 at various total pressures from 100 to 760 mm. Hg. These results show that, within this pressure range, an increase in flame speed is produced by increasing the mole fraction of water vapor at least as far as saturation at 25° C., and that the rate of this increase is greater the higher the pressure. It is evident that water vapor plays an important part in the explosive oxidation of CO; the need for further experimental evidence as to the nature of its action is indicated.

Report No. 532, The Soap-Bubble Method of Studying the Combustion of Mixtures of CO and O_2, by Ernest F. Fiock and Carl H. Roeder. 14 pages, tables, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1935. Price five cents.

This investigation, carried out at the National Bureau of Standards at the request and with the financial assistance of the National Advisory Committee

for Aeronautics, is a detailed description of the soap-bubble, or constant-pressure method as applied to the explosive oxidation of CO.

A series of values of the speed of flame in space in various mixtures of CO and O_2 containing a constant percentage of water vapor was obtained by the constant-volume method. These results served as a guide in the perfection of the soap-bubble method.

The latter has been refined to a degree which makes possible precise determinations of the speeds of flame in space and relative to the active gases, of expansion ratio, and of temperatures attained in mixtures of CO, O_2, and H_2O. Results for this system of gases are reported over a wide range of mixture ratios, and a comparison with previous results by the same method is included.

PUBLICATIONS RECEIVED.

Electrotechnik. Ein Leitfaden für Studium und Praxis, Herausgegeben von Dr.-Ing. G. Bolz, Dr.-Ing. F. Moeller, Dipl.-Ing. Th. Werr. Band II, Teil 3 und 4. Gleich- und Wechselstrommaschinen, von F. Moeller und Th. Werr. 125 pages, illustrations, diagrams, 16.5 × 24 cms. Leipzig und Berlin, B. G. Teubner, 1935. Price 10 R.M.

Energie Übertragung und- Umwandlung mit Welchselstrom. Einheitliche Theorie der Leitungen, Transformatoren und Maschinen von Paul Werners. 204 pages, illustrations, 16.5 × 24.5 cms. Leipzig und Berlin, B. G. Teubner, 1935. Price 25 R.M.

Mechanical and Electrical Equipment for Buildings, by Charles Merrick Gay and Charles De van Fawcett. 429 pages, illustrations, 15.5 × 23.5 cms. New York, John Wiley & Sons, 1935. Price $5.00.

The Optical Basis of the Theory of Valency, by R. de L. Kronig, Ph.D. 246 pages, tables, illustrations, 14.5 × 22.5 cms. New York, The Macmillan Company. Cambridge, University Press, 1935. Price $4.50.

Commémoration du Centenaire de la Mort de Joseph-Nicéphore Niépce, Inventeur de la Photographie, sous le haut Patronage de M. le Président de la République, avec la Présidence effective de M. le Ministre de l'Education nationale, Manifestations organisées à Chalon-sur-Saône en Juin 1933. 78 pages, illustrations, 16.5 × 25.5 cms. Paris, Societe Francaise de Photographie et de Cinematographie.

Physics, by Erich Hausmann, E.E., and Edgar P. Slack, S.B. 776 pages, illustrations, tables, 14.5 × 22 cms. New York, D. Van Nostrand Co., Inc., 1935. Price $4.00.

The Theory of Atomic Spectra, by E. U. Condon, Ph.D., and G. H. Shortley, Ph.D. 441 pages, illustrations, 18 × 27 cms. New York, The Macmillan Company. Cambridge, University Press, 1935. Price $11.00.

Inventing the Ship, A Study of the Inventions Made in Her History between Floating Log and Rotorship, by S. C. Gilfillan, Ph.D. 294 pages, illustrations 14.5 × 20.5 cms. Chicago, Follett Publishing Company, 1935. Price $2.50.

Experimental Physics, by Edwin Morrison, M.S., and S. Elisabeth Morrison,

B.S. 235 pages, illustrations, tables, 22 × 28 cms. Philadelphia, P. Blakiston's Son & Co., Inc. 1935. Price $2.00.

Structure et Propriétés des Noyaux Atomiques. Rapports et Discussions du Septieme Conseil de Physique Tenu a Bruxelles du 22 au 29 Octobre 1933, sous les auspices de l'Institut International de Physique Solvay. 353 pages, Illustrations, 16.5 × 25 cms. Paris, Gauthier-Villars, 1934. Price 75 francs.

U. S. Coast and Geodetic Survey, Special Publication No. 194, Manual of Traverse Computation of the Lambert Grid, by Oscar S. Adams and Charles N. Claire. 235 pages, tables, diagrams, 15 × 23 cms. Washington, Government Printing Office, 1935. Price twenty cents.

Canada Dominion Bureau of Statistics, Census of Industry. Report on the Distilled Liquor Industry in Canada, 1934. 12 pages, 22 × 28 cms. Advance Report on the Textile Industries of Canada, 1933. 78 pages, tables, 22 × 28 cms. Ottawa, King's Printer, 1935.

U. S. Bureau of Mines, Statistical Appendix to Minerals Yearbook, 1934. Gold and Silver, by J. P. Dunlop, 434 pages, tables, 15 × 23 cms. Washington, Government Printing Office, 1935. Price five cents. Bulletin 386, Quarry Accidents in the United States during the Calendar Year 1933. 62 pages, tables, 15 × 23 cms. Washington, Government Printing Office, 1935. Price ten cents.

Bell Telephone Laboratories, Monograph B-854, Probability in Engineering, by E. C. Molina, 13 pages, tables. B-855, Vacuum Tubes for Generating Frequencies above 100 Megacycles, by C. E. Fay and A. L. Samuel, 14 pages, tables, illustrations. B-862, Ferromagnetic Distortion of a Two-Frequency Wave, by Robert M. Kalb and William R. Bennett, 38 pages, diagrams. B-863, The Detection of Frequency Modulated Waves, by J. G. Chaffee, 24 pages, diagrams. B-864, Internal Dissipation in Solids for Small Cyclic Strains, by R. L. Wegel and H. Walther, 17 pages, diagrams. B-866, Diurnal and Seasonal Variations in the Ionosphere, by J. P. Schafer and W. M. Goodall, 12 pages, diagrams. B-867, Direct-Current Amplifier Circuits for Use with the Electrometer Tube, by D. B. Penick, 6 pages, diagrams. Wide Band Transmission in Sheathed Conductors, by O. B. Blackwell, 11 pages. Electrical Communication: Past, Present, Future, by Frank B. Jewett, 33 pages, maps. 9 pamphlets, 15.5 × 23 cms. New York, Bell Laboratories, 1935.

National Advisory Committee for Aeronautics, Technical Notes: No. 534, Principal Effects of Axial Load on Moment-Distribution Analysis of Rigid Structures, by Benjamin Wylie James, 46 pages, diagrams, tables. No. 535, The Effect of Depth of Step on the Water Performance of a Flying-Boat Hull Model N.A.C.A. Model 11-C, by Joe W. Bell, 8 pages, figures, No. 536, The 6-Foot-4-Inch Wind Tunnel at the Washington Navy Yard, by G. L. Desmond and J. A. McCrary, 13 pages, figures. No. 537, A Preliminary Determination of Normal Accelerations on Racing Airplanes, by N. F. Scudder and H. W. Kirschbaum, 8 pages, tables. No. 538, A General Tank Test of N.A.C.A. Model 11-C Flying-Boat Hull, Including the Effect of Changing the Plan Form of the Step, by John R. Dawson, 7 pages, figures. No. 539, Aerodynamic Characteristics of Several Airfoils of Low Aspect Ratio, by C. H. Zimmerman, 7 pages, figures. 6 pamphlets, 20 × 26 cms. Washington, Committee, 1935.

CURRENT TOPICS.

Concerning the Toad.—It is apparently in the best interests of self-preservation for the toad to make itself as distasteful as possible. According to biologists of the United States Department of Agriculture, a milky poisonous fluid is secreted in a pair of large, glandular masses behind the toad's eyes. Any animal having the temerity to bite the toad will find that this secretion produces a bitter astringent taste. However, this defensive equipment does not protect the toad from snakes, skunks or birds of prey. Skunks in eating toads usually roll them about roughly with their paws until the poisonous secretion has been discharged and rubbed off.

C.

The Use of Water to Extinguish Oil Fires.—A. L. BROWN (*Quarterly*) of the National Fire Protection Association, July 1935). A statement that water can be used for the effective control of oil fires may sound revolutionary, at least to the practical fire fighter. Careful consideration however of the action of a burning inflammable liquid will point to the fact that its exposed surface at least is at, or above, the temperature corresponding to the fire point. If the burning liquid can be cooled below this temperature, the fire will no longer burn. It is possible then, that water applied rapidly and effectively might reduce the temperature of a burning oil to nearly the temperature of the water, extinguishing the fire. The "Mulsifyre" system discharges sprays of water forcibly on the surface of the burning liquid. It is a development of Mather and Platt, Ltd. of Manchester, England. The underlying principle here is that of converting a flammable liquid into one which will not burn by emulsifying it with water. A strong spray of water is applied by a projector; the discharge varies from about 15 to 20 gallons at 40 lbs. water pressure, which is considered the lowest pressure giving an effective spray. Possibly it is allowable to contemplate an application of water in such fine drops that it does not settle or penetrate the flammable liquid rapidly, thereby leaving essentially a water layer on top. The Grinnel Company of Providence, R. I., has made an installation to protect an asphalt saturating tank near Bridgeport, Conn., and is now working on systems to protect oil-filled transformers and high-pressure lubricating oil systems at public service power plants in New York.

R. H. O.

Free Cutting Aluminum Alloys for Screw Machine Work.—(*Iron Age*, Vol. 136, No. 4) "Free cutting" is a term coined to denote the rapid breaking up of cut scrap so that it falls away from the machined piece, leaving the metal surface clean and not cluttered for the oncoming tool. The Aluminum Co. of America has supplied data that show that this material has now been added to the list of "free cutting" metals. The problem to solve consisted of the proper alloying of aluminum, and the solution was found with the discovery that the simultaneous presence of small concentrations of a multiplicity of elements was much more efficient in producing satisfactory machineability than any one element alone. From a number of elements found to be satisfactory, lead and bismuth were chosen to supply multiplicity because it was further found that these elements had relatively small effect on the other properties of the alloy. The effect of copper on machineability was also kept in mind and its concentration was placed at about the maximum for the satisfactory hot working of large ingots. There are now available at least two alloys in large scale production; one with magnesium content, one without, in two tempers, so that there is a choice of four different materials. Tool wear on free cutting aluminum alloys appears to be slightly higher than on free cutting brass, but is much lower than that obtained with steel or duraluminum screw machine stock.

R. H. O.

New Automatic Adjustable Blade Turbine.—(*Power Plant Engineering*, Vol. XXXIX, No. 8.) There is now under construction by the Kanawha Valley Electric Co. at its Marmet Plant in West Virginia, a hydraulic development, involving a 7600 hp. adjustable blade turbine of new design with an experimental runner. Heretofore this runner has been built only in model sizes for experimental purposes in the laboratory of the Newport News Shipbuilding and Dry Dock Co. The runner vanes are pivoted on roller bearings and connected through gears and racks to a dashpot in the runner hub. Each vane is pivoted slightly above the center of pressure so that the water flow creates a hydraulic moment which tends at all times to open or increase the pitch of the vanes. This moment is opposed by a device which creates a reactive moment that tends at all times to close or decrease the pitch of the vanes. The reactive device may be pre-compressed springs or any one of several other devices. The runner vanes therefore remain in a balanced position with changing load which in turn governs the adjustment of the wicket gates. The relation between

the position of the vanes and the position of the wicket gates is not fixed, but varies with the head and speed. The vanes open wide when the turbine is started and this gives maximum starting torque. As the turbine approaches normal speed with no load on the unit, the vanes move to the closed position, after which they move to the position required by the load.

R. H. O.

Heat Transmission in Boilers.—What appears to be a very important development in heat transmission in boilers is described by P. ST. GEORGE KIRKE in the *Gas Journal*, Vol. 211, No. 3766. After considerable experimenting the author produced a new boiler tube, the object being to enable many more molecules of gas that are near the tube walls to arrive there and transmit their heat to the wall themselves. In practice this has resulted in an actual acceleration of heat transmission of 150 per cent. In other words, the heating surface required has been reduced by 60 per cent. and the heat transmission per unit of heating surface has been increased in the ratio of 2½ to 1. "Sinuflo" is a registered trade mark. The tube as illustrated is bent in curves similar to a sine wave, to the degree that a brush or scraper on a shaft can easily be poked through. Due to the shape of the tube the gases are always flowing towards the tube wall, first towards one side then towards the other. A town gas fired "Sinuflo" boiler fitted with vertical tubes 2 in. O/D × 5 ft. long, submerged in water 4 ft. was independently tested by a gas company and the thermal efficiency found by deducting the known losses from 100 per cent. was 91.25 per cent. This boiler is entirely independent of chimney draught. There is a separate gas jet opposite each tube and combustion takes place in the tube, so that the bottom tube plate is not subject to any flame temperature at all, which enables them to be made for pressures as high as 750 lbs. per sq. in.

R. H. O.

Airplanes Solve Alaskan Mining Problems.—The opening up of new mining fields in rugged out of the way places no longer presents the old-time difficulty of the transportation of supplies and materials. CLARENCE WM. POY, Manager of the Big Four Mine, Inc., Valdez, Alaska, describes airplane freighting methods in *Mining and Metallurgy*, Vol. 16, No. 344, which eliminated to a great extent transportation handicaps. Most of the supplies are delivered in winter on ski-equipped planes to the mine high up in the mountains. A Fairchild monoplane is used with a load capacity of about 800 lbs.

It is constructed so that 8 ft. to 9 ft. lengths of lumber, pipes and other supplies can be handled easily. The skis are especially constructed and adapted to local conditions of deep snow. They are also rigged to obtain maximum efficiency in turning around in small spaces, and for quick take-offs on short landing spaces. A long, narrow ski, with a long, sloping turnup both fore and aft of the ski pedestal has been found most efficient. The bottom of the ski is lined with No. 22 gage black iron which gives considerable strength to the ski, and does not cling to damp snow. Extra strong rubber shock cord is attached on the nose of the airplane to the nose of the ski to help raise the skis out of the snow on the take-offs. The ski is constructed so that the axle is 37 per cent. of the distance from the heel of the ski to the nose of the ski. During 1934, more than 60 tons of mining machinery and supplies were transported to the mining properties by airplanes without any damage or loss. These supplies included everything necessary for the development and putting into production of two mines.

R. H. O.

Insulated Cars for Shipment of Dry Ice.—(*Railway Age*, Vol. 99, No. 6.) Five such cars of the A. A. R. Class Rc have recently been built by the American Car and Foundry Co. for the Liquid Carbonic Corporation, Chicago. They are 45 ft. long and of steel construction throughout. The entire bottom half consists of a solid bed of insulation surrounding the steel bins provided to hold the dry ice. Up to a height of 8 ft. above the rail the width is 10 ft. 8 in. From this point the sides are tapered in to a width of 8 ft. 11 in. at the eaves which are 13 ft. 3 in. above the rail. These dimensions were adopted in order to provide maximum lading and insulation for the container bins, sufficient inside head room for the men engaged in loading, and at the same time to provide adequate clearance for tunnels, etc. The load limit of the cars is 79,500 lbs. The actual loads will be approx. 30 tons. The insulated sides are provided with ventilating louvers near the eaves. The central side doors are of the sliding type and extend only half way down the sides. Insulating materials are cork and balsam wool and fir wood sheathing is used for outside of the steel bins. The tops of the bins are open, but when in service are covered with an 8 in. flexible blanket of kapok. These cars will transport dry ice from manufacturing to distributing points with relatively small losses.

R. H. O.

A Refrigerated Slot Machine.—C. A. RODMAN of Detroit, Mich., describes his development in part, for the Cincinnati Ball Crank Co.,

of a new package ice cream vending machine in *Refrigerating Engineering*, Vol. 30, No. 1. Previous machines of this type have experienced trouble, due to leakage and to the opening and closing of the machine at the time of the delivery of the merchandise, enough moisture entering the machine to form ice in the merchandise compartment, coin and delivery mechanism. Not only has this difficulty been overcome, but the final design has a mechanism that will handle five different kinds or flavors of ice cream that can be selected at will. The machine is refrigerated by cold air. A special evaporator is located in a separate compartment, with a blower for circulation of the cold air to and through the merchandise compartment, and back to the evaporator chamber. This evaporator is so designed as to hold a 90 per cent. relative humidity in the merchandise compartment, but will not permit excess moisture to collect there. The machine operates between 0° and 5° and will go for fifteen days without defrosting. It is cooled by a 1/8 h.p. hermetic compressor. At the time of defrosting, the water from the defrost is re-evaporated by the condenser heat, so that there are no drips or drains or pans of any kind. The machine pulls 150 watts at 110 V. when it is in operation, which is about 60 per cent. of the time, at 90° F. room temperature. It weighs 90 lbs.

R. H. O.

Ice Solves a Gas Distribution Problem.—The golden rule in public utility practice is to maintain uninterrupted service. At times this becomes difficult in gas distribution, especially when both normal and emergency repairs must be made on high pressure pipe lines. Mr. I. H. Witt in *Western Gas*, Vol. XI No. 8, describes a new process to overcome this difficulty. The initial operation consists of clamping to the pipe at the point at which it is desired to shut off the flow a suitable pipe saddle containing a gas main cock and stuffing box above the cock. The main is next drilled and tapped, working through the stuffing box and cock, preventing any escape of gas. Next a cold pack is placed on each side of the saddle, consisting of cracked ice and salt or dry ice in canvas wrapped around the pipe. When the pipe is sufficiently chilled, a mixture of finely shaved ice and water is forced into the gas main from a specially constructed pump, the lower end of which is inserted into the main through the stuffing box and cock. The cold packs cause the shaved ice to freeze into a solid gas-tight ice block across the whole cross section of the main. When it is desired to reëstablish the flow of gas the cold packs can be removed and heat applied to the outside of the main. Then the other apparatus can be removed, plugging the drilled hole with a brass plug. There is no escape of gas during the

whole process. A by-pass around the area of operation allows it to be done without interruption to service. The method and apparatus are patented.

R. H. O.

A New Carrier-Current Coupling Capacitator.—By means of coupling capacitators connected to high voltage power transmission lines, high frequency carrier currents in different frequency channels may be transmitted over power lines for supplementary services such as relaying, control, and communication. E. D. EBY, in *Electrical Engineering*, Vol. 54, No. 8, describes new coupling capacitators to be used on the Boulder Dam power transmission system of the City of Los Angeles bureau of power and light. A higher line voltage will be employed than has previously been used in this country. Several advantages favored small series connected units in contrast with a single large unit capacitator for this voltage. A survey of existing operating voltages showed the prominence of 69,138 and 230 kv. circuits, suggesting units rated at 69 kv. From a review of the 60 cycle and impulse characteristics of line apparatus and line insulation for these circuit voltages, it was found that a 69 kv. capacitator unit in series combinations of 2, 3 and 4 units, respectively, would coördinate satisfactorily with the insulation requirements of 138,230 and 287 kv. circuits. Since these capacitators were intended to serve the double purpose of carrier current coupling and potential coupling, it was necessary to consider the output demanded by both classes of service. It was found that the carrier-current demand for the 287,000 volt circuit would be provided for by a capacitance of 0.00075 microfarad. Thus, with 4 units in series, the unit capacity would be 0.003 microfarad. This capacitance would also provide adequate output for potential devices—150 volt amperes.

R. H. O.

An Automobile Fuel That Supplies Both Refrigeration and Power.—*Food Industries* for August, 1935 has an unsigned article describing a combination fuel-refrigeration system for food trucks. In such an arrangement propane, a gas at ordinary temperatures, cools the truck body and takes the place of gasoline. Propane is closely related chemically to gasoline, the former being classed as one of the lower paraffin hydrocarbons, while the latter, a volatile liquid, is composed of paraffins more complicated in structure. As a matter of economy, propane is supplied in its liquid form in pressure cylinders that are interchangeable. Since it boils at minus 40° F. or lower, it is only necessary to release the pressure to bring

it once more to its vapor form and suitable as an engine fuel when mixed with air.

When the propane evaporates it absorbs heat. Its refrigeration properties are thus utilized for cooling the truck body of the vehicle being driven by the propane vapor as fuel. The setup is relatively simple. Air, in the double-walled spaced of the body, is circulated by means of a fan over the cold coils through which the gaseous propane passes on its way to the carburetor. Although the B.t.u. content is somewhat under that of gasoline on a volumetric basis, it is much dryer and so its efficiency is raised to on par with the latter. Propane has an octane rating of 125 and when the compression ratio of the engine is raised, its efficiency is still further increased.

C.

Organic Inhibitors for Iron and Steel.—There are numerous occasions when iron and steel must come in contact with corrosive acids although it is desirable that the metal shall not be attacked. This can be accomplished by adding to the acid solutions, an inhibitor, a substance which by a mechanism not yet completely understood, prevents dissolution of the metal but does not interfere with any action the acid may have upon the metal oxides (rust) or upon other susceptible materials.

In the August, 1935, issue of *Chemical Industries* H. A. LEVEY describes some of the more recent findings concerning the protective powers of organic inhibitors. These latter have come into prominence comparatively recently, although mention has been made of arsenic as an inorganic inhibitor as far back as 1905. The first effective organic inhibitor was reported in 1920 by Griffin who found that the addition of small amounts of formaldehyde greatly reduced the solvent action of hydrochloric acid on steel. Once a systematic investigation was undertaken it was found that a large number of organic substances exerted an inhibiting effect to greater and less degrees. Thus, when classified in generic groups, these showed an increasing inhibiting action in the order named: sulfonated oils, nitrogen base oils from gas works, paper pulp waste liquor, sulfonated animal matter, low grade wheat flour, nitrogen bases in acid and finally, coal-tar nitrogen bases in acid. These last are from 95 per cent. to 98 per cent. effective.

According to the author of the article, "the inhibitors most extensively used commercially today are the crude nitrogen bases obtained by extracting coal-tar oils or similar materials with diluted sulfuric acid. These acid extracts are commonly sold and used as such with little, if any, further purification or concentration. They

usually are more or less heterogeneous and variable in composition and contain . . . considerable quantities of inert or even objectionable impurities." The remarkable influence of an efficient inhibitor is better appreciated when it is realized that one-hundredth of a mol of phenylacridine exerts 92 per cent. inhibiting power and a like proportion of 3,6 dimethyl 1–2–7 diethyl aminoacridine hydrochloride is 99.8 per cent. effective.

Naturally, the most extensive use of inhibitors will be found in pickling operations where the acid is used to dissolve away scale and oxides and minimum attack on the metal is desired. The same principle is applied in dairies where it is sometimes necessary to dissolve from equipment a dense lime scale using acid with an inhibitor to protect the metal parts. A more recent application comprises a process for the reactivation of dormant or exhausted oil wells. This involves the pumping of hydrochloric acid down the pipes where it attacks the calcareous formations. An inhibitor prevents the acid from dissolving away the steel casings and iron pumps but does not interfere with destruction of the calcite.

C.

Making Bread from Hydrogen Peroxide instead of Yeast. (*Chemical Industries*, August, 1935.) At a recent meeting of the American Association of Cereal Chemists L. H. BAILEY and J. A. LECLERC described the use of hydrogen peroxide to replace yeast in breadmaking. It was found that the hydrogen peroxide bread had slightly more crust than the yeast bread, and its moisture content was a little lower. The peroxide bread contained approximately three times as much sugar and soluble nitrogen as the material made from yeast. It likewise contained twice as much water-soluble solids and slightly less soluble starch. The percentages of fat, ash, and total nitrogen in both breads were the same. The peroxide bread possessed as good a grain and better texture than the corresponding yeast bread. The loaf of the former was fully as large as the latter but, and here's the catch, the flavor of the peroxide bread was poor, as the flavor of ordinary bread is largely due to yeast activity.

C.

Cathode-Ray Oscillograph Engine Indicator.—(*Engineering*, Vol. CXL, No. 3632.) The cathode-ray oscillograph has the unique freedom from inertia required for the measurement of pressure changes of high frequency, and the application of the instrument to indicate the process of combustion in an engine cylinder has engaged the attention of the research staff of the Anglo-American

Oil Co. for the past five years. The development work has now reached the stage that diagrams can be taken free from spurious vibration and mechanical or electrical lag at any engine speed, and even when detonation is in progress. It has been used satisfactorily in the laboratories to determine the combustion characteristics of various fuels in spark ignition engines, and for the determination of ignition characteristics in Diesel engines.

A particular advantage of the instrument lies in the fact that any desired portion of the engine cycle can be selected and magnified on the time base, thus enabling a very close study to be made of such phenomena as ignition lag in Diesel engines and detonation in petrol engines, both of which occur during such a small crank-angle that analysis is practically impossible on the conventional pressure-time diagram. The instrument may also be adjusted to obtain diagrams on a stroke base.

Visual observation of diagrams can be made conveniently, or can be recorded photographically. Such an instrument will doubtless become a factor in the further development of the internal combustion engine.

R. H. O.

The Bus of 1940.—A very interesting forecast of the type of bus the public will be riding in in 1940 is given in a paper by R. A. HAUER before the recent meeting of the Midwest Transit Association in Denver, an abstract of which appears in the *Transit Journal* for August, 1935. Many important changes will be evident. Fuel costs per ton mile will be less. Engines will have higher thermal efficiency, and improved transmissions will permit the engine to operate at more nearly their economy peak than at present. It is possible that other and more economical fuels may be available. The diesel is showing more promise daily and the development of butane-propane fuel may possibly provide an answer. Either force may parallel developments on gasoline engines. In 1940 practically all new buses will have rear-mounted engines and there will doubtless have been developments along somewhat unconventional lines as to cylinder arrangement, such as division at the middle, with the clutch drive take-off from the center of the crankshaft; a swashplate engine; or a tight V arrangement. The bus of 1940 will be definitely lighter for the same capacity and performance. It will have a low floor, no ramps or steps inside. Long distance buses will be as thoroughly air conditioned as trains, but it is doubtful if air conditioning will be attempted in city type buses. Heaters may be located above the windshield with fresh air blown in over them, foul air outlets being located at the rear and near the floor, and of

such capacity that there will always be a slight pressure of air within the bus when all doors and windows are closed. Much will be done to simplify driving.

R. H. O.

Effect of Wind on Temperature Rise of Transmission Line Conductors.—In the June number of the JNL. OF FRANKLIN INSTITUTE on page 785 there is mention of a method of melting sleet on transmission lines by applying short circuit current to raise the temperature of the conductor. H. GANDEFROY and K. H. LIPPITT in the *General Electric Review* for August state that another method suggested is to pass a low power-factor current around the circuit to raise the conductor temperature. While some work has already been done in determining the temperature rise of transmission cables, the effect of wind has not been studied thoroughly. Thesis work was performed under direction of H. B. Dwight and R. H. Smith at M. I. T. in 1934 on 211,600 cir. mil standard copper cable, 19 strands and a 350,000 cir. mil cable of 37 strands using wind velocities ranging from 5 to 50 miles per hour in the 7.5 ft. wind tunnel at M. I. T. From the test results it may be deducted that: (1) the temperature rise is directly proportional to some power of the watts per square inch, and inversely proportional to some power of the wind velocity; (2) from both practical and theoretical standpoints, the temperature rise seems to be directly proportional to the watts per square inch, making the exponent 1; (3) the exponent of the wind velocity varies with the cable tested. It was shown that heat transfer is greater when turbulence is present and a stranded cable would produce much turbulence because of the irregularities of its surface. The relation between temperature rise and wind velocity therefore seems dependent upon the roughness of the cable. Just to what extent is not known, but it seems to be considerable.

R. H. O.

Auto-Stop for Railway Crossings.—(*Ry. Age*, Vol. 99, No. 8.) A system of barriers, known as the Auto-stop, which rise in the highway approaching the tracks, have been installed at a crossing of a single-track main line of the Chicago and North Western in Des Moines, Iowa. The crossing is located on tangent level track four miles from the station where train speeds range from 30 m.p.h. for freight trains to 50 m.p.h. for passenger trains. The Auto-stop consists of two barriers, located in the highway 75 ft. on each side of the track. The barriers raise, when a train is approaching, to prevent the movement of highway vehicles over the tracks. A barrier is constructed in two sections 10 ft. long, each of which

extends half way across the highway. The top plate, normally level with the surface of the highway, and the face of the barrier are formed from ⅞ in. steel plate. Track circuits in connection with the automatic block signals control the raising and lowering of the barriers. There are two levels at which the barriers are raised. With the warning or hesitation position, at the four inch height, a vehicle striking the barrier from either direction forces it down to the level of the pavement, the barrier returning to this position. When in the fully raised position, 9½ in. high, the section on the right side of the highway is locked. However, the section on the left side of the highway which rises to a full height of only 6 in. is not locked in the raised position, so that if the barriers are raised while a vehicle is on the track side the driver is not trapped on the tracks, for, as the car is driven up the slanting slope of the left section from the track side, the barrier is depressed to the pavement and then rises again when the car passes. The installation was officially placed in service on April 22.

R. H. O.

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